The Brattle Group

Estimating the Cost of Equity for Regulated Companies

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TABLE OF CONTENTS

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EXECUTIVE SUMMARY

In this report, we discuss the models available for estimating the cost of equity for the purpose of the Natural Gas Rules in Australia. Given that the new Rule 87 requires relevant estimation methods, financial models and market data to be considered, as well as the "prevailing conditions in the market for equity funds", this report focuses on the characteristics of the various models, how they perform under various market conditions, and therefore how to assign weight to a method, model or other data based on prevailing market or industry conditions. Further, the report finds that practitioners, regulators, and textbooks commonly look to several models or data sources before reaching a conclusion on the cost of equity.

All models have relative strengths and weaknesses, with the result that there is no one model that is the most suitable for estimating the cost of equity at any given time or for any given company. As our colleague and MIT professor Stewart Myers has put it eloquently "Use more than one model when you can. Because estimating the opportunity cost of capital is difficult, only a fool throws away useful information." This report provides a set of guidelines that can be used in deciding which models should have more weight than others under different market, industry, or company-specific circumstances.

The focus of the report is on the key characteristics of the various cost of equity estimation methods available for a decision maker and circumstances under which each method may be more or less suitable. It is imperative that the choice of model(s) and their implementation take into account the prevailing economic conditions, industry specifics as well as characteristics of the firm for which the cost of equity is being determined, because, according to the circumstances, each model can show bias. We therefore emphasize that there is no single or formulaic approach to estimating the cost of equity. Evidence from academics, practitioners and regulators alike agree that a mechanistic reliance on a single model, without regard to changing market or industry conditions, may deliver spurious results.

The different models should be applied to a set of comparable firms, rather than the single firm for which the cost of equity is to be determined, because all methods for estimating the cost of

equity introduce significant noise or uncertainty. Applying the models to a set of comparator firms generates a range of cost of equity estimates for each model. Consideration of prevailing economic conditions, industry specifics, and characteristics of the firm for which the cost of equity is to be determined should go to the weight that is put on each model in deriving an overall reasonable range for the cost of equity.

For example, a dividend growth model might have more weight and the Sharpe–Lintner CAPM less weight when (as currently) interest rates on government bonds are unusually low. Conversely, a dividend growth model might have less weight, and the CAPM more weight, in a sector where growth forecasts are considered to be less reliable. In addition, empirical results from the Sharpe–Lintner CAPM suggest that results may be biased for firms with beta significantly different from one. In addition to the traditional Sharpe-Lintner CAPM and dividend growth models, the report also discusses other models such as the Black CAPM, the Fama-French model, the Consumption CAPM, and the Arbitrage Pricing Theory. We also touch upon new developments in implementing the dividend discount model and on other data and evidence that is sometimes used in combination with the models mentioned above.

Once a reasonable range for the cost of equity has been identified, selecting a point within that range is a matter of judgment, but that judgment can be guided by considering the riskiness of the firm at hand relative to the riskiness of the comparable firms used to generate the cost of equity estimates. Only non-diversifiable risks should be included—for example, variation in demand, which might be more highly correlated with general economic growth for a utility with significant industrial load than for a utility serving mostly residential customers.

I.   INTRODUCTION

The Australian Energy Market Commission recently changed the rules that guide the regulation of pipelines (and other regulated entities) in Australia.   The Australian Pipeline Industry Association (APIA) has therefore asked *The Brattle Group* (*Brattle*) to review the methods that are currently used or could be used to estimate the cost of equity capital for the purposes of the National Gas Rules in Australia. As part of this exercise, the APIA has asked us to review how academics, practitioners and regulators worldwide think models should be used, and how they have been used in determining the cost of equity for regulated entities. Thus, in this report, we discuss examples of regulatory approaches in the U.S., Canada and the U.K. where regulators have considered a number of methods for estimating the cost of equity capital, and have determined the optimal use of these multiple evidence sources in order to provide greater confidence in their results. The report also includes a discussion of the recommendations of academics and practitioners with regards to the use of several cost of equity estimation models.

The report focuses on the new Rule 87 and the new *allowed rate of return objective,* which, in order to be achieved, requires that "regard must be had to relevant estimation methods, financial models, market data and other evidence"[1] in determining the overall rate of return, and that "regard must be had to the prevailing conditions in the market for equity funds"[2] in determining the cost of equity component of the overall rate of return. We therefore focus on introducing a broad set of methods for cost of equity estimation, the risk positioning of a company relative to the industry or other companies, and methods relied upon by regulators and practitioners around the globe.

Section II provides some background for cost of equity estimation. *Section III* focuses on the evolution, theoretical underpinnings, and characteristics of various cost of equity estimation methods including (a) the Sharpe-Lintner Capital Asset Pricing Model (CAPM), (b) variations of the CAPM such as the Empirical CAPM (ECAPM) and the Consumption-Based CAPM, (c) the Fama-French Three-Factor Model, (d) the Arbitrage Pricing Theory, (e) Dividend Discount

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[1]   Rule 87, s.5a.

[2]   Rule 87, s.7.

Models including both Single-Stage and Multi-Stage models, and (f) Other Models including the so-called Risk Premium method, Residual Income Valuation model, Ibbotson's Build-up method, the Comparable Earnings model, Market-to-Book and Earnings Multiples approaches. We note that the above is not intended to be an exhaustive list of models that regulators or practitioners could feasibly rely upon in determining the cost of equity. We also note that as finance evolves, new estimation methods, financial models, market data and other evidence may become available that could be informative for the purpose of estimating the cost of equity. *Section IV* discusses implementation issues, summarizes the characteristics of the various cost of equity estimation methods, and discusses how to use the models under different market conditions. Additionally, this section includes a description of how to position the target entity relative to a sample based on its relative risk.

II. METHODS, FINANCIAL MODELS, MARKET DATA AND OTHER EVIDENCE USED TO ESTIMATE THE COST OF EQUITY CAPITAL

A. INTRODUCTION

To determine the cost of capital, one must evaluate the cost of equity, the cost of debt (possibly both long-term and short-term) and the capital structure of the company subject to regulation. This report focuses on the estimation of the cost of equity component of a regulated entity's cost of capital.

To determine the cost of equity for a specific utility, decision makers typically look at a range of evidence presented to them. In the case of regulators, they commonly review expert evidence, models and other information presented by experts, the utility and other stakeholders, and also evidence that the regulator itself generates. Ultimately, a degree of judgment is used to arrive at a final determination having considered this evidence. The evidence considered might include different financial models which are used to extract estimates of the cost of equity for similar utilities from market data (stock prices). It might also include estimates from models that take equity analyst forecasts as inputs. For example, three regulators, the Alberta Utilities Commission (AUC), the Ontario Energy Board (OEB), and the U.S. Surface Transportation Board (STB), recently reviewed their cost of equity estimation approach. These three regulators noted that each methodology has its own strengths and weaknesses and subsequently decided to

rely on more than one model or approach to determine the cost of equity.[3] We further note here that in discussing the characteristics of each model or practice, we are pointing to advantages or disadvantages of the models assuming they will inform the ultimate decision, but we do not expect any one model to be the only piece of evidence considered and used by either regulators or practitioners in determining the cost of equity.

This report describes a number of models that can be used to inform the regulator's judgment in determining the cost of equity. It also discusses the views of academics and practitioners with regards to the determination of the cost of equity from multiple estimation models.

Below, we describe methodologies that regulators and practitioners use in Australia, Canada, Europe, the U.K., and the U.S., as well as some more recent methods that have been proposed, albeit it is not clear from the record the extent to which regulators have used these methods. It is important to realize that in many jurisdictions the regulator does not look to a single model, but considers all the evidence in front of it and then makes a decision. In North America, where the consideration of more than one model and possibly other evidence is common, the ultimate decision is often not explicit about the weight assigned to each model or other pieces of evidence.[4]

B. THE USE OF MODELS FOR COST OF CAPITAL ESTIMATION

1. Context

The National Gas Rules set the framework for how the AER (and the ERAWA) determine access arrangements for covered gas pipelines, including the rate of return on capital which is a component of the charges paid by pipeline customers. We understand that the regulators are

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[3]   Alberta Utilities Commission, Decision 2011-474, p. 27-28, Ontario Energy Board, EB-2009-084, p. 38, Surface Transportation Board, Ex Parte 664 (Sub-No. 1), pp. 3-5.

[4]   There are exceptions to this rule such as the Federal Energy Regulatory Commission and the Surface Transportation Board in the U.S., and the Canadian Transportation Agency. However, most U.S. state and Canadian federal and provincial regulators do not have a specified cost of equity estimation method. Instead, they commonly hear evidence from a number of different parties on cost of equity (often including regulatory staff). Based on this information the regulator then makes its decision.

currently developing guidelines as to how the rate of return provisions of the NGR will be applied in future determinations.

The NGR state that "… the rate of return for a service provider is to be commensurate with the efficient financing costs of a benchmark efficient entity with a similar degree of risk… ".[5] In addition, the NGR require that "[I]n determining the allowed rate of return, regard must be had to: (a) relevant estimation methods, financial models, market data and other evidence;…"[6] and that "[i]n estimating the return on equity under subrule (6), regard must be had to the prevailing conditions in the market for equity funds."[7]

In this report, we describe the estimation methods, financial models, market data and other evidence that may be relevant for setting the cost of equity in future access arrangement determinations in Australia.

a) The cost of capital

The cost of capital is a key parameter in regulatory settings, because it contributes to determining the return to the company's investors. Defined as *the expected rate of return in capital markets on alternative investments of equivalent risk*, it is the expected rate of return investors require based on the risk-return alternatives available in competitive capital markets. Stated differently, the cost of capital is a type of opportunity cost: it represents the rate of return that investors could expect to earn elsewhere without bearing more risk.[8, 9]

While the details of energy network regulation are different in different jurisdictions, regulators are in many jurisdictions required to set a cost of capital which provides investors in rate-regulated entities a reasonable opportunity to earn a return on their investment equal to the opportunity cost of capital.

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[5]   Rule 87(3).

[6]   Rule 87(5).

[7]   Rule 87(7).

[8]   "Expected" is used in the statistical sense: the mean of the distribution of possible outcomes. The terms "expect" and "expected" in this Report, as in the definition of the cost of capital itself, refer to the probability-weighted average over all possible outcomes.

[9]   The cost of capital is a characteristic of the investment itself, not the investor.

In the U.K., the Gas Act 1986 requires the regulator to have regard to "the need to secure that licence holders are able to finance the[ir] activities.…"[10] Ofgem has also said:

> In setting price controls, we are required to have regard to the ability of efficient network companies to secure financing in a timely way and at a reasonable cost in order to facilitate the delivery of their regulatory obligations.[11]

In Canada, the National Energy Board has explained the "fair return standard" as follows:

> The Board is of the view that the fair return standard can be articulated by having reference to three particular requirements. Specifically, a fair or reasonable return on capital should:
> - be comparable to the return available from the application of the invested capital to other enterprises of like risk (the comparable investment standard);
> - enable the financial integrity of the regulated enterprise to be maintained (the financial integrity standard); and
> - permit incremental capital to be attracted to the enterprise on reasonable terms and conditions (the capital attraction standard).[12]

Finally, in the U.S., the starting point for the Federal Energy Regulatory Commission's approach to determining the cost of equity is Supreme Court precedent, which states that:

> the return to the equity owner should be commensurate with the return on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital.[13]

While these legal standards are differently worded, a common thread is that regulated entities are allowed to earn a return that is comparable to that of other enterprises of similar risks and which enables the regulated entity to finance its operations. The legal standards in North America and Europe are not specific about how to accomplish the goal(s).

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[10]   Gas Act 1986, s. 4AA(2)(b).

[11]   *RIIO-T1: Final Proposals for National Grid Electricity Transmission and National Grid Gas*, Ofgem (December 2012), paragraph 4.6.

[12]   RH-2-2004, p. 17. See also the Supreme Court of Canada's decision in Northwestern Utilities Limited v. City of Edmonton [1929] S.C.R. 186.

[13]   *FPC v. Hope Natural Gas Co.*, 320 U.S. 591 (1944). *Bluefield Water Works &*

Improvement Co. v. Public Service Comm'n, 262 U.S. 679 (1923), cited in FERC policy statement on the *Composition of Proxy Groups for Determining Gas and Oil Pipeline Return on Equity*, April 17 2008, p. 2.

b) *What should we expect from models?*

It is useful to recognize explicitly at the outset that models are imperfect. All are simplifications of reality, and this is especially true of financial models. Simplification, however, is also what makes them useful. By filtering out various complexities, a model can illuminate the underlying relationships and structures that are otherwise obscured. After all, while a perfect scale model representation of the city might be highly accurate, it would make a poor road map. It is therefore imperative that regulators and other users of the models use sound judgment when implementing and using the models — there is no one model or set of models that are perfect.

The gap between financial models and reality can sometimes be quite significant (as was painfully demonstrated by the recent financial crisis). There is no single, widely accepted, best pricing model to estimate the cost of capital — just as there is still no consensus on some fundamental issues, such as the degree to which markets are efficient. Analysts have a host of potential models at their disposal, and it must be acknowledged that cost of capital estimation continues to require the exercise of judgment. Practitioners, regulators, as well as textbooks therefore often recommend that the "best practice" for ensuring robustness is to look at a totality of information.[14] These practitioners, regulators and texts therefore use or present a variety of methodologies that may be applicable for the determination of the cost of equity in a specific circumstance.

While no model is perfect, there are certain features that make models more useful from a regulatory perspective. For example, it is desirable to have models and methods that i) are consistent with the goal being pursued, ii) are transparent, iii) produce consistent results, iv) are robust to small deviations or sampling error, v) are as simple as possible (while maintaining reliability), vi) can be replicated by others (*e.g.,* data is widely available), and vii) recognize the regulatory context and legislative requirements in which the regulatory body operates. Clearly different models will satisfy these criteria to differing degrees, and different models may be better suited to different regulatory jurisdictions.

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[14] See, for example, the Ontario Energy Board's EB-2009-084 decision, December 2009, the U.S. Surface Transportation Board's Ex. Parte 664 (Sub-No. 1) decision, January 2009, Morningstar *Ibbotson Cost of Capital 2012 Yearbook*, and Roger A. Morin*, New Regulatory Finance*, Public Utilities Reports Inc., 2006, Chapter 15.

For example, the CAPM and the Dividend Discount Model (DDM) both are transparent and developed from economic theory. Their results can be replicated easily, since the data required are widely available from many public sources. However, the implementation of the CAPM and DDM requires a number of subjective decisions – decisions which can be hotly contested and can lead to significantly different results. The CAPM, for instance, relies on a risk-free rate that is currently driven unusually low by the recent flight to quality and the easing of monetary policy. The model also requires an estimate of the market risk premium, which may pose difficulties in times of high market volatility.

The single-stage DDM is especially sensitive to the growth rate estimates used, which can vary widely among analysts and over time, contradicting the underlying assumption of growth stability inherent in this model. The variability in growth rates and stock prices may increase when industries are in transition, making the reliability of the DDM more questionable in such periods. In addition, it has become more common to distribute cash to shareholders in a form other than dividends. For example, regulated entities in both the U.S. and the U.K. have had share buyback programs that substantially affected the number of shares, and these are not captured in the standard DDM.[15] Some of the growth rate problems in the DDM are alleviated by the reliance on a multi-stage version of the model as done by, for example, *The Brattle Group*, Morningstar *Ibbotson Cost of Capital Yearbook*, and the U.S. Surface Transportation Board (STB).[16]

Similar problems arise in other models that inherently rely on data for a sample of companies and data for economic phenomena that may be changing quickly; the latter is especially true for models such as the Fama-French, where the reliance on three risk factors can lead to highly variable results across time. As a result, no single model is ideal and the implementation of any model necessarily requires choices that involve subjective judgments. Therefore, it is important to look to the totality of relevant information available from methods, models, market data and

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[15] See, for example, National Grid Share Buyback Programme and Spectra Energy Corp's 2008 form 10-K.

[16] *The Brattle Group* is a consulting firm, Morningstar is a commercial provider of data and the STB is a U.S. federal regulator.

other evidence. The relative strengths and weaknesses of the various cost of equity estimation models are outlined in further detail in *Section III* of this report.

c)   *Model stability and robustness*

For an estimation model used to determine the cost of equity, stability and robustness over time are desirable unless economic conditions have truly changed. Stability means that cost of capital estimates done in similar economic environments should be similar, not only period-to-period but also company-to-company within a comparable sample. Robustness is meant here as the ability of a model to estimate the cost of capital across different economic conditions.

In general, all of the models discussed here have characteristics that make them more or less suited to one economic environment versus another. As such, all individual models can be, and often are, subject to some instability over time. For example, the currently very low government bond yields lead to very low cost of equity estimates using the CAPM — sometimes less than the costs of debt of investment-grade companies! During the early 2000s, the DDM was subject to substantial criticism due to allegations of analysts' optimism bias. Similarly, the risk premium model[17] has produced very different results in times of high and low inflation that did not necessarily reflect the true cost of capital. Thus, estimates at any given point of time may seem too high or too low, and it is important to understand whether the estimated figures are driven by actual changes in the systematic risk of the regulated entities, or by something else (*e.g.,* data irregularities). It is for these reasons that regulators in the U.S. and Canada often rely on and analysts recommend relying on the results from at least two estimation models.[18]

A notable example of a regulator that has acknowledged the difficulty in relying on only one model or method is the U.S. Surface Transportation Board. The STB in 1982 started to rely on a single-stage DDM to determine the cost of equity for U.S. railroads. However, in 2006, the shippers on the railroads complained that the estimated cost of equity was out of line with reality,

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[17]   The risk premium used in the risk premium model is different from the market risk premium used in the CAPM. The model is frequently used in U.S. regulatory proceedings.

[18]   See, for example, U.S. Surface Transportation Board, Ex Parte 664 (Sub-No. 1), served January 28, 2009; Mississippi Power, Performance Evaluation Plan, Rate Schedule 'PEP-5', November 9, 2009 (http://www.mississippipower.com/pricing/pdf/pep-5.pdf); Ontario Energy Board, EB-2009-0084, Report of the Board on the Cost of Capital for Ontario's Regulated Utilities, Issued December 11, 2009.

because forecasted growth rates for railroad companies were substantially higher than the economy-wide forecasted growth. The shippers argued successfully that such high growth rates could not be sustained forever as assumed by the single-stage DDM, and the STB thus initiated a rulemaking proceeding to review and eventually determine how to set the allowed cost of equity going forward. Following several years of expert submissions and proceedings, the STB decided to rely on an equally-weighted average of the Sharpe-Lintner Capital Asset Pricing Model and a specific version of the multi-stage DDM. In doing so, the STB concluded:

> if our exploration of this issue has revealed nothing else, it has shown that there is no single simple or correct way to estimate the cost of equity for the railroad industry, and countless reasonable options are available. Both the CAPM and the multi-stage DCF [DDM] models we propose to use have their own strengths and weaknesses, and both take different paths to estimate the same illusory figure. By using an average of the results produced by both models, we harness the strengths of both models while minimizing their respective weaknesses. The result should be a stable yet precise estimate of the cost of equity that we can use in future regulatory proceedings and to gauge the financial health of the railroad industry.[19]

2. Risk-Return Tradeoff

At its most basic level, an asset (security) is a claim to a stream of future (risky) cash flows and sometimes with potential rights to exert some control over those flows. Financial markets allow investors to exchange these claims, and therefore risks. Through trade, investors are able to create different packages of risks and returns than could be achieved by holding individual securities (or fixed packages of securities), and investors can change their risk exposure over time. Because investors are assumed to be risk-averse, they evaluate the universe of risky investments on the basis of a risk-return trade-off. Investors can only be induced to hold a riskier investment if they expect to earn a higher rate of return on that investment. The essential tradeoff between risk and the cost of capital is depicted in Figure 1 below.

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[19] *U.S. Surface Transportation Board, Ex Parte 664 (Sub-No. 1)*, served January 28, 2009, p. 15.
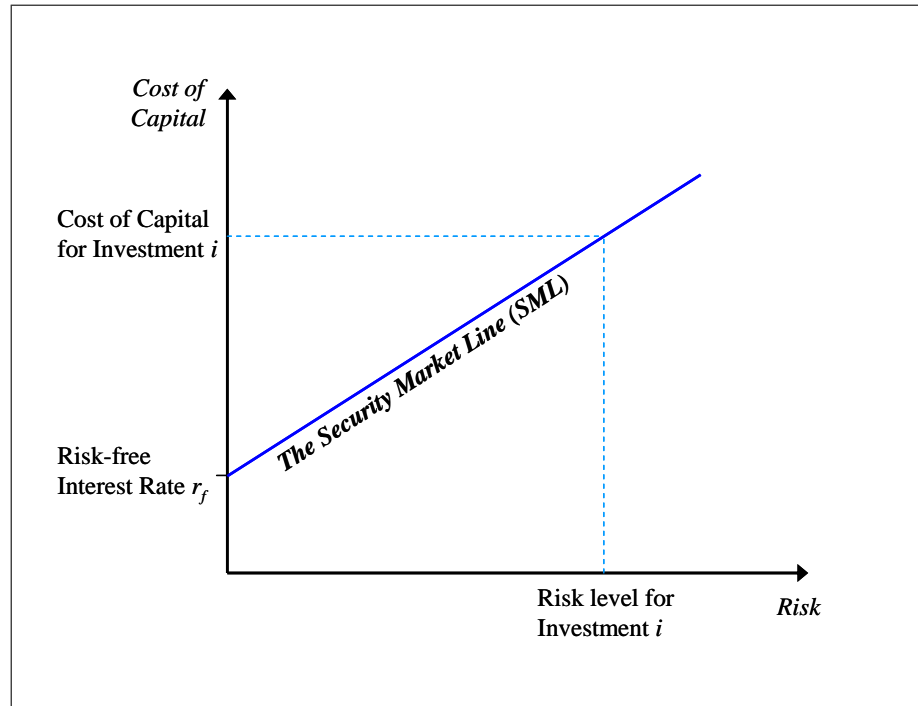
Figure 1: Security Market Line

III. COST OF EQUITY ESTIMATION MODELS

A. SHARPE-LINTNER CAPITAL ASSET PRICING MODEL

One of the most common pricing models used in business valuation and regulatory jurisdictions is the Sharpe-Lintner CAPM, which in its simplest form is depicted in Figure 2 below.
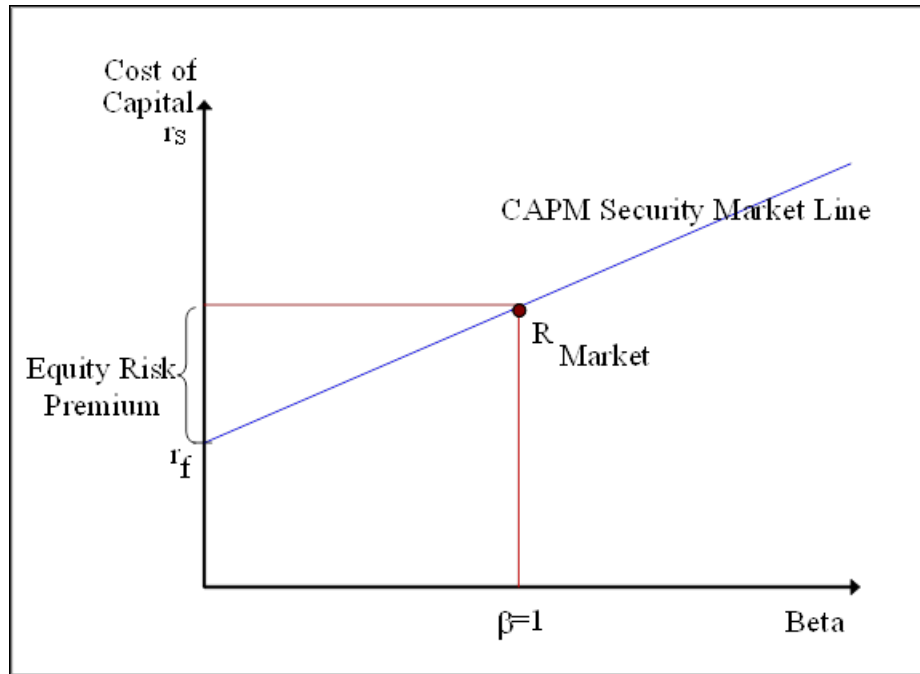
Figure 2: Capital Asset Pricing Model

Thus, in the world in which the CAPM holds, the expected cost of (equity) capital for an investment is a function of the risk-free rate, a measure of systematic risk (beta), and an expected market risk premium (MRP):[20]

$$E(r_S - r_f) = \beta_S \times E(r_M - r_f) \tag{1}$$

where r_S is the cost of capital for investment S; r_M is the return on the market portfolio, r_f is the risk-free rate, and β_S is the measure of systematic risk for the investment S. The $(r_M - r_f)$ term is known as the market risk premium (MRP),[21] and β_S measures the response of the stock S to systematic risk. Re-arranging this equation produces the CAPM's formula for the cost of (equity) capital of a traded asset:

$$r_S - r_f = \beta_S \times MRP \tag{2}$$

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[20]   While the CAPM model frequently is applied to equity capital, it applies to all assets.

[21]   We note that some European regulators use the term Equity Risk Premium (ERP) instead of MRP.

To implement the CAPM, it is necessary to determine the risk-free rate, r_f, and to estimate the MRP and beta, β_S.

1. Evolution of the CAPM

The CAPM was developed as a theoretical equilibrium model and fits with the intuition of a risk-return tradeoff. The development of the CAPM signaled the first time that economists were able to quantify risk and the reward for bearing it. Under the CAPM, the expected return of an asset must be linearly related to the covariance of its return with the return of the market portfolio.[22]

Markowitz (1959)[23] first laid the groundwork for the CAPM. In his seminal research, he expressed the investor's portfolio selection problem in terms of expected return and variance of return. He argued that investors would optimally hold a mean-variance efficient portfolio, that is, a portfolio with the highest expected return for a given level of variance. Sharpe (1964)[24] and Lintner (1965)[25] built on Markowitz's work to develop economy-wide implications. They showed that if investors have homogeneous expectations and optimally hold mean-variance efficient portfolios, then, in the absence of market frictions, the portfolio of all invested wealth, or the market portfolio, will itself be a mean-variance efficient portfolio. This is the heart of the Sharpe-Lintner CAPM. The standard CAPM equation (as expressed in Equation (2)) is a direct implication of this statement.

The Sharpe-Lintner CAPM assumes unrestricted lending and borrowing at a risk-free rate of interest. In the absence of a risk-free asset, Black (1972)[26] derived a more general version of the CAPM which did not rely on this potentially problematic assumption. In this version, known as the Black CAPM, the expected return of an asset in excess of the "zero-beta" return is linearly

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[22] For a basic introduction to risk-return models, see R.A. Brealey, S.C. Myers, and F. Allen, Principles of Corporate Finance, 10ed, 2011 (Brealey, Myers & Allen (2011), pp. 192-203.

[23] H. Markowitz, "Portfolio Selection: Efficient Diversification of Investments," 1959, John Wiley, New York.

[24] W. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* 19, 1964, pp. 425-442.

[25] J. Lintner, "The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics 47, 1965, pp. 13-37.

[26] F. Black, "Capital Market Equilibrium with Restricted Borrowing," *Journal of Business* 45, 1972, pp. 444-455.

related to its market beta. In essence, the return on the risk-free asset in Equation (2) above is substituted with a return on a zero-beta portfolio associated with the market portfolio. This zero-beta portfolio is defined to be the portfolio that has the minimum variance of all portfolios uncorrelated with the market portfolio. The empirical implementation of the Black CAPM is often referred to as the Empirical CAPM or ECAPM.

Empirical tests of the Sharpe-Lintner CAPM have focused on three implications of equation (2): (i) The intercept is zero; (ii) The market beta completely captures the cross-sectional variation of expected excess returns; and (iii) The market risk premium is positive.

There is substantial literature on empirical tests of the CAPM since its development in the 1960s, with mixed results. Black, Jensen and Scholes (1972)[27], Fama and Macbeth (1973),[28] and Blume and Friend (1973)[29] found empirical evidence to be consistent with the mean-variance efficiency of the market portfolio. However, Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) identified a fundamental challenge to the CAPM; namely that low-beta stocks have higher average returns than predicted by the CAPM, and high-beta stocks lower average returns. In other words, the empirical estimates are consistent with pivoting the Security Market Line (SML) around beta = 1 compared to the Sharpe-Lintner CAPM. This suggests that the cost of capital for regulated companies, which often have a beta less than one, will be underestimated by the traditional CAPM.[30]

Several subsequent studies confirmed the robustness of this result and proposed explanations revolving around market frictions, such as different borrowing and lending rates, and the role of

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[27] F. Black, M.C. Jensen, and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," *Studies in the Theory of Capital Markets*, Praeger Publishers, 1972, pp. 79-121.

[28] E. Fama and J. Macbeth, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy* 81, 1973, pp. 607-636.

[29] M. Blume and I. Friend, "A New Look at the Capital Asset Pricing Model," *Journal of Finance* 28, 1973, pp. 19-33.

[30] Implementing a long-run version of the CAPM which uses (annualized) long-horizon returns (*e.g.*, with long bond rates as risk-free rate) generally produces a flatter SML than obtained by using short-rates, due to the general presence of an upward sloping yield curve. While this partially compensates for the empirically observed flattening, it is not sufficient to explain all of the observed flattening of the SML. That is, even implementations that utilize a long-run risk-free interest rate require a further, albeit smaller, adjustment to match the empirical SML.

taxes. Nevertheless, the empirical evidence suggested significant movement in the SML, often flattening, to the point that Fama and French (1992) found a zero slope in the empirical SML.[31] Fama and French (1992, 1993[32]) in turn suggested that factors other than the risk relative to the market, such as size and book-to-market value ratios (among others) were significant in explaining the observed SML. Fama and French found that firms with high book-to-market ratios and small size have higher average returns than is predicted by the standard CAPM, and vice versa. Their work culminated in the model now known as the Fama-French three-factor model.

The Fama-French papers cited above continued in the vein of the so-called "anomalies" literature that had arisen in the late 1970s. These anomalies can be thought of as firm characteristics that provide incremental explanatory power for the sample's mean returns beyond the market. Earlier anomalies included the price-earnings ratio effect (first reported by Basu (1977)[33]) and the detection of the size effect (Banz (1981)[34]). For example, Basu found that firms with low price-earnings ratios have higher sample returns than those predicted by the standard CAPM. The price-earnings ratio and size anomalies are at least partially related, as low price-earnings-ratio firms tend to be small.

The Empirical CAPM (ECAPM), described further in the section below on variations of the standard CAPM, is an alternative method of correcting for the empirical flattening of the SML. The ECAPM can be viewed from the positive school of thought as a practical adjustment that can be made to measure the cost of capital. It can be applied without knowing the "cause" of the increased intercept and decreased slope of the SML relative to the Sharpe-Lintner CAPM.

To sum up, there has been a wealth of statistical evidence contradicting the Sharpe-Lintner CAPM over the past 40 years or so and controversy remains about how the evidence should be

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[31] E.F. Fama and K.R. French, "The Cross-Section of Stock Expected Returns," *Journal of Finance* 47, 1992, pp. 427-465.

[32] E.F. Fama and K.R. French, "Common risk factors in the returns on stocks and bonds," *Journal of Financial Economics* 33, 1993, pp. 3-56.

[33] S. Basu, "The Investment Performance of Common Stocks in Relation to Their Price to Earnings Ratios: A Test of the Efficient Market Hypothesis," *Journal of Finance* 32, 1977, pp. 663-682.

[34] R. Banz, "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics* 9, 1981, pp. 3-18.

interpreted. Some argue that the standard CAPM should be replaced by multifactor models with several sources of risk, such as the Fama-French model. Others argue that evidence against the CAPM is overstated due to potential mis-measurement of the market portfolio, data mining or sample selection biases. One further key deficiency in the CAPM is that it is a static model which ignores consumption decisions, and treats asset prices as being determined by the portfolio choices of investors who have preferences defined over wealth one period in the future. Implicitly, these models assume that investors consume all their wealth after one period or at least that wealth uniquely determines consumption. This assumption does not match with reality. Therefore, to make the model more realistic, intertemporal equilibrium asset pricing models have been developed that model consumption and portfolio choices simultaneously. An example of such a model is the consumption-based CAPM, which is described further in *Section III.B.2* below.

2.   CAPM Implementation Issues

Fundamentally, an analyst using the CAPM must determine three parameters to implement the model: the risk-free rate (r_f), the MRP, and the asset's beta (β_S) as shown in Equation (2) above. Through the determination (or estimation) of the parameters on the right-hand side of Equation (2), the analyst obtains an estimate of the cost of equity, r_S.

It is common to choose (i) a forecasted yield on government bonds (as is often done in Canada), (ii) a current measure of local government bond yields (a common practice in the U.S.), or (iii) a regional or global measure of the current yield on government bonds (*e.g.*, the Netherlands).

Like the risk-free rate, the choice of market proxy is local, regional, or global. The choice of risk-free rate and market index should be consistent, so the cost of equity is estimated as either a local, regional, or global figure.

For many years it was common to estimate the MRP from an arithmetic average of historical realized MRPs, measured as the long-term excess of market returns over the risk-free rate in the country or region of interest. European decision makers have in recent years often looked to the study of Dimson, Marsh, and Staunton to determine the MRP, while many in the U.S. commonly

look to evidence from Morningstar (formerly Ibbotson).[35] Some decision makers and analysts also look to either forecasted MRPs or survey results.[36] The estimation of the MRP remains controversial and the resulting cost of equity estimates generated by the standard CAPM are sensitive to the choice of MRP.

3.   Characteristics of the CAPM

While the strengths and weaknesses of the CAPM inherently depend on its exact implementation, the following are some generic strengths:

- The model is transparent, well-documented and relies on economic theory.

- Data needed for the model are readily available if applied to companies with a reasonable trading history in well-developed markets. It is therefore also auditable.

- The model is sensitive to economic conditions through risk-free rates and market performance, as well as to changes in companies' systematic risk.

Among the weaknesses of the CAPM are the following:

- The model is very sensitive to developments in the risk-free rate that may reflect monetary policy rather than economic conditions.

- The model is sensitive to different estimation procedures for the MRP.

- Because beta estimates rely on historical data, there may be a delay in incorporating changes in systematic risk. MRP estimates based on historical data are also backward-looking.

- The model may downward bias cost of equity estimates for low-beta stocks and vice versa (see section on ECAPM below).

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[35] Texts such as Morningstar, *Ibbotson SBBI 2012 Yearbook*, p. 55-56 recommends to use the income return rather than total return or yield as the risk-free rate. The income return consists of the coupon payment divided by the bond price rather than the total return as this is the true risk-free component of the bond return. Capital gains or losses carry risk.

[36] For examples, see Bank of England, "Financial Stability Report," June 2012, Chart 1.11 and P. Fernandez, J. Aguirreamolla and L. Corres (2013), "Market Risk Premium used in 82 countries in 2012: a survey with 7,192 answers," IESE Business School, University of Navarra, SSRN 2084213.

- The model incorporates only one source of risk (the market), and therefore does not reflect the effects of, for e.g., consumption or economic growth, technological or regulatory risks.

- The CAPM is a static model and therefore ignores the dynamics of investment behavior and hedging.

- The model is based on the assumption that all investors optimally hold well-diversified portfolios and therefore only care about systematic risks. This assumption does not necessarily hold, however, when investor expectations about returns and investment opportunities are heterogeneous.

Because the model was developed as a generic approach to determining the cost of capital for companies, it does not specifically take industry factors or the context in which it is being used into account. However, the CAPM is a well-founded and commonly used model that relies primarily on readily available information. It may be less stable than ideal because changes in interest rates affect the risk-free rate and market volatility affects the beta estimates. Furthermore, determination of which sample companies to rely upon and the MRP remains controversial.

The CAPM has been widely used for a long period of time for a variety of reasons. The primary reason for the model's widespread use is its solid economic foundation, making it taught in every introductory finance class. The model is also relatively simple to implement. Most market-based models that have been developed since the CAPM take the CAPM as their point of departure to generalize the model. Also, academic researchers have not found any one alternative to the model that is easily applied in practice.

B. VARIATIONS ON THE CAPM

1. The Empirical CAPM

As described above, the ECAPM is one way of correcting for the empirical flattening of the Security Market Line (SML). Specifically, the ECAPM directly adjusts the CAPM SML by a parameter, alpha, that can be controlled for sensitivities, *etc.* Formally, the ECAPM relation is given by Equation (3) below:

$$r_s = r_f + \alpha + \beta_s \times (MRP - \alpha) \tag{3}$$

where α is the "alpha" adjustment of the risk-return line, a constant, and the other symbols are as defined above. The alpha adjustment has the effect of increasing the intercept but reducing the slope of the SML, which results in a security market line that more closely matches the results of empirical tests.
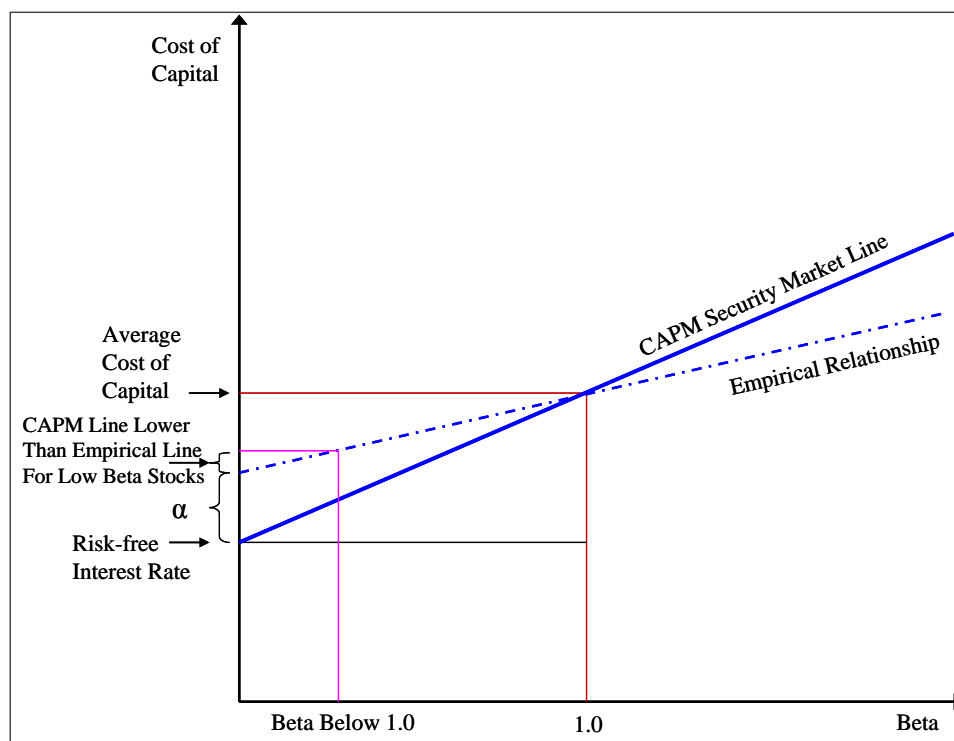
Figure 3: The Empirical Security Market Line

The academic literature has estimated a fairly wide range of alpha parameters, using primarily U.S. data, of approximately 1 to 7 percent.[37] While this is a rather large range, much of the variation between studies arises from differences in methodology and time periods so that the alpha estimates are not strictly comparable. The ECAPM is included among the models relied upon by some decision makers and experts including U.S. state and Canadian provincial regulators.[38]

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[37] See Appendix A for details.

[38] The Mississippi Public Service Commission in the U.S. and the Alberta Utilities Commission in Canada have included the ECAPM as one of the models used to determine the cost of equity.

2. The Consumption-Based CAPM

The Consumption CAPM is an example of an intertemporal equilibrium model. This model aggregates investors into a single representative agent and considers a changing investment opportunity set over time, unlike the static standard CAPM. The representative agent is assumed to derive utility from the aggregate consumption of the economy. In this model, the stochastic discount factor, (defined such that the expected product of any asset return with the stochastic discount factor is equal to one), is equal to the intertemporal marginal rate of substitution for the representative agent.[39] Through mathematical equations, (the so-called Euler equations), asset returns and consumption can be linked. Using this setup, the model explains the risk premia on assets using the covariance between their returns and the intertemporal aggregate consumption marginal rate of substitution. As a result, the consumption-based pricing model can help explain the observed phenomenon of predictable variations in asset risk premia over time, and expands the risk-return relation to allow for a time-varying relationship between a stock's risk and return.

An important feature of the consumption model is that the expected conditional risk premium on an asset is related to its predicted conditional volatility. In particular, the relationship between a stock's risk premium and its conditional volatility could be positive or negative, depending on the extent to which the stock is an intertemporal hedge against shocks to the marginal utility of consumption. Furthermore, hedging assets have volatility patterns that could lead to expected rates of return lower than the risk-free rate. Note that this would generally not be the case for public utility stocks, since they are not viewed as defensive stocks.

Several versions of the consumption-based CAPM have been developed. In one of the more applicable versions, the addition of assumptions about the preferences of investors allows the model to explain the risk premia on assets through their covariance with consumption growth, so that the model, to a degree, can explain variations in the excess returns of risky assets over time. Other versions of the model allow time-varying investor risk aversion to explain predictable movements in risk premia.

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[39] This is equal to the discounted ratio of marginal utilities for the representative agent in two successive periods.

In a regulatory setting, the consumption CAPM can be used to either project the expected risk premium over the risk-free rate or verify the relied-upon market risk premium. The model has not commonly been used in a regulatory setting, but a recent implementation of Ahern, *et al.* (2012)[40] was developed explicitly to estimate the cost of equity for regulated entities. The description below therefore focuses on this version of the model.

The Ahern model is estimated using a so-called GARCH-in-mean (GARCH-M) model, which unlike the Sharpe-Lintner CAPM allows for the stock returns to depend on a volatility (variance) measure. In particular, the GARCH-M specification is such that the expected risk premium on a stock is a linear function of its conditional volatility. In this model, the parameter of interest, α, which represents the linear relationship between the risk premium on the stock and the conditional volatility in the GARCH-M model, can be translated into the following implication of the theoretical asset pricing model described above:

$$\alpha = -\frac{vol_t[M_{t+1}]}{E_t[M_{t+1}]} corr_t[M_{t+1}, R_{t+1}] \tag{4}$$

where R_{t+1} is the expected total return on the public utility stock index or individual utility stock, and M_{t+1} is the stochastic discount factor (SDF), *i.e.*, the (aggregate) consumption intertemporal marginal rate of substitution. The equation above implies that the coefficient on volatility will be positive (*i.e.*, returns and conditional volatility will be positively correlated) if the conditional correlation between the SDF and the asset return is negative, *i.e.*, if the stock is not a hedging asset.

Ahern, *et al.* (2012) estimate the conditional risk-return model using monthly total returns from January 1928 to December 2007 on the S&P Public Utilities stock index, and the monthly Moody's public utility Aa, A, and Baa yields for the cost of debt. The authors then compare the model's performance with the performance of, for example, the Sharpe-Lintner CAPM. The estimates of the cost of common equity from the model are similar to the CAPM values and

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[40] P.A. Ahern, F.J. Hanley, R.A. Michelfelder, "New Approach to Estimating the Cost of Common Equity Capital for Public Utilities," *Journal of Regulatory Economics*, 2012 (Ahern, *et al.* 2012)

appear to be stable and consistent over time. Thus, the empirical implementation of the theoretical model resulted in cost of equity estimates that appeared to be within a range of reasonableness. The model has been presented in some U.S. regulatory jurisdictions but regulatory decisions based on the model are either still pending or it is not clear how the regulator used the information. Ahern, *et al.* conclude that the consumption-based asset pricing model "should be used in combination with other cost of common equity pricing models as additional information in the development of a cost of common equity capital recommendation".[41]

3. Characteristics of CAPM Variations

As for the CAPM, the strengths and weaknesses of the variations discussed above depend on the implementation of the models. However, some strengths of the models are:

- Both the ECAPM and the Consumption CAPM allow for empirically observed phenomena to be modeled:

  ▸ The ECAPM recognizes the flatter-than-predicted-by-CAPM Security Market Line.

  ▸ The Consumption-CAPM allows for the expected risk premium to vary with asset and investor characteristics, such as conditional volatility and risk aversion.

- Data needed for the models are usually available if applied to companies with a reasonable trading history in well-developed markets. The models are therefore also auditable.

- The models are sensitive to economic conditions. The Consumption-CAPM considers more factors than does the CAPM.

Among the weaknesses of the models are the following:

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[41]   Ahern, *et al.* (2012), p. 17.

- The ECAPM has not been tested extensively outside the U.S. or in recent market conditions.

- The Consumption CAPM relies on the use of more data than does the CAPM and requires a refined estimation process, which makes it less accessible to a broader audience.

C.   THE FAMA-FRENCH THREE-FACTOR MODEL

The Fama-French model holds that the expected return of a security is described by an augmented CAPM relationship:

$$E(r_S - r_f) = \beta_S \cdot E(r_M - r_f) + s_S \cdot E(SMB) + h_S \cdot E(HML) \tag{5}$$

where $E(r_M - r_f)$ is the market risk premium (MRP) as used in the CAPM, SMB is the difference in returns between small companies and big companies ("*Small Minus Big*"), and HML is the difference in returns between securities of firms with a high book-to-market equity ratio and a low one (*"High Minus Low"*). The factor loadings s_S and h_S represent security S's "holding" of each of these risk factors, which is to say they are the regression coefficients of r_S on each of the factors.

Evolution of the Fama-French Three-Factor Model

Fama and French (1992) was the last influential paper in a series of academic research into the placement of the empirical SML relative to the theoretical CAPM. Controlling for firm size, the authors found no relationship between the market and expected return (zero beta). Stated differently, any explanatory power that the market beta in the CAPM might have is absorbed by using size to explain the cross-sectional variation in returns. Fama and French interpreted this to mean that market beta (and by extension the CAPM) had zero explanatory power for expected returns. Moreover, they found that all of the variation in returns that were (in other research) associated with size, earnings/price ratios, book-to-market equity ratios, and leverage, could be captured by size and the book-to-market equity ratio alone. Fama and French (1993) ultimately settled on a three-factor model that brought the market return back into the model (size, book-to-market ratio, and market return). Their 1993 paper found that this model explained 90 percent of

the variations in the cross-section of returns, and it has since become known as the Fama-French three-factor model.

From an empirical perspective, the Fama-French model is an alternative to the ECAPM – one should not employ a Fama-French model with an alpha adjustment (Equation (3)). However, the interpretation of the findings of Fama and French has been critiqued by many academics as the size and book-to-market factors may proxy for other phenomena.[42]

Standard Implementation:

The SMB factor and HML factor are typically created following Fama & French's (1993) approach. Specifically, at each point in time one allocates each firm into the small or big category, according to whether its market cap is in the top or bottom half of all firms considered. The firms in each half are then value-weighted to form two portfolios: small firms and big firms. The difference in realized returns between each of these portfolios is then taken as the SMB realization in that period. Creation of the HML series is similar, but firms are allocated to the "high" category if their book-to-market ratio is in the top 30th percentile and to the "low" category if their book-to-market ratio is in the bottom 30th percentile. These two time series can then be used to estimate the average SMB and HML, as well as the factor loadings for a given security; *i.e.*, the factors in the regression version of Equation (5), β_S, s_S, and h_S are estimated.

As a practical matter, the SMB and HML factors can be obtained free of charge from Professor Kenneth French's website,[43] where he maintains a database of the factors for regional areas such as Asia-Pacific, Europe, and North America.

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[42] For a discussion of this critique, see, for example, Black, F., "Beta and return," *Journal of Portfolio Management* 20, 1993, pp. 8-18; A.C. MacKinlay, "Multifactor Models Do Not Explain Deviations from the CAPM," *Journal of Financial Economics* 38, 1995, pp. 3-28; A. Lo and A.C. MacKinlay, "Data-Snooping Biases in Tests of Financial Asset Pricing Models," *Review of Financial Studies* 3, 1990, pp. 431-467; Fama, E. and K.R. French, "Size and Book-to-Market Factors in Earnings and Returns," *Journal of Finance* 50, 1995, pp. 131-155; and Fama, E., and K.R. French, "Industry costs of equity," *Journal of Financial Economics* 43(2), 1997, pp. 153-193.

[43] The website is located at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Regulatory Use

The Fama-French model has been submitted in Australia, North America, and the U.K.[44] While U.S. decisions are only rarely explicit about how evidence was weighted, we are not aware of a U.S. decision that primarily relied on the Fama-French model. However, the U.K. Competition Commission used the model to determine whether a small company premium should be included in the cost of capital.[45] The Régie de l'énergie in Québec considered the Fama-French approach and found that the model had not been sufficiently examined to date to be used as a basis for setting the rate of return for a gas distributor.[46]

Characteristics of the Fama-French Three-Factor Model

Many of the Fama-French model characteristics are similar to those of the CAPM. It relies on a risk-free rate and an estimate of the market risk premium, so like the CAPM it is sensitive to developments in risk-free rates. Like the ECAPM, the Fama-French model captures the empirical observation that the Security Market Line predicted by the CAPM is too steep. The Fama-French model has two additional factors, which vary over time and therefore add to the variations in the cost of equity estimates over time.

D. ARBITRAGE PRICING THEORY

The Arbitrage Pricing Theory (APT) was developed by Ross (1976a, 1976b)[47] as a multifactor alternative to the CAPM. The model is a theoretical approach to explaining the cross-section of returns with additional factors beyond the standard market portfolio in the Sharpe-Lintner CAPM. It is a one-period model in which all investors believe the stochastic properties of capital assets' returns are consistent with a factor structure. Assuming equilibrium prices offer no arbitrage opportunities, the expected returns on these capital assets are approximately linearly

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[44]   See, for example, Jemena Gas Networks (NSW) Ltd - Initial response to the draft decision - Appendix 5.2 - NERA: Cost of Equity – Fama-French Model; California Public Utilities Commission, "Decision 07-12-049," December 20, 2007; and U.K. Competition Commission, "Market Investigation into Supply of Bulk Liquefied Petroleum Gas for Domestic Use: Provisional Findings Report," August 2005, Appendix K.

[45]   See, for example, U.K. Competition Commission, "Market Investigation into Supply of Bulk Liquefied Petroleum Gas for Domestic Use: Provisional Findings Report," August 2005, Appendix K.

[46]   Régie de l'énergie, Décision D-2007-116, Gaz Métropolitain, pp. 23-24.

[47]   S.A. Ross, "Options and Efficiency*," Quarterly Journal of Economics* 90, 1976, pp. 75-89 and S.A. Ross, "The Arbitrage Theory of Capital Asset Pricing*," Journal of Economic Theory* 13, 1976, pp. 341-360.

related to the factor loadings. The factor loadings are proportional to the returns' covariances with the factors - much like in the CAPM.[48]

The empirical specification of the model is

$$E(r_S) = \beta_1 \cdot E(Factor1) + \beta_2 \cdot E(Factor2) + ... + \beta_N \cdot E(FactorN) \tag{6}$$

The APT is a generalization of the standard CAPM in that it allows for multiple risk factors and does not require the identification of the market portfolio. However, the theoretical APT only provides an approximate relation between expected asset returns and a combination of factors. Therefore, testability of the model depends on imposing several additional assumptions on the conditional distribution of returns. For example, exact factor pricing holds in an equilibrium intertemporal asset pricing framework. In this general model specification, the market portfolio is one pricing factor as in the standard CAPM, and additional factors arise from investors' need to hedge uncertainty about future investment opportunities. These factors can be specified as traded portfolios of assets, or macroeconomic variables that reflect the systematic risks of the economy, such as industrial production growth, changes in bond yield spreads or unanticipated inflation.

The key difference between factor specification in the APT versus the Fama-French model described above, is that the factors in the APT are theoretically motivated as hedging variables that capture economy-wide non-diversifiable risks, whereas the factors in the Fama-French model are empirically motivated, and are instead selected based on observing the firm characteristics that best explain the cross-section of returns over a specific sample period.

E. DIVIDEND DISCOUNT MODEL

Although there are several versions of the Dividend Discount Model (DDM), all versions determine today's stock price as a sum of discounted cash flows that are expected to accrue to shareholders. Assuming that dividends are the only type of cash payment to shareholders, the pricing formula becomes:

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[48] For a brief introduction, see Gur Huberman, "Arbitrage Pricing Theory," in *The New Palgrave: Finance*, eds. J. Eatwell, M. Milgate, and P. Newman, 1989, pp. 72-80.

$$P_t = \frac{E_t(D_1)}{(1 + r_S)} + \frac{E_t(D_2)}{(1 + r_S)^2} + \frac{E_t(D_3)}{(1 + r_S)^3} + \cdots \tag{7}$$

where "P_t" is the market price of the stock; "D_i" is the dividend cash flow at the end of period i; "r_S" is the cost of capital of asset/security S (as before); and the sum is into the infinite future.[49] The formula above says that the current stock price is equal to the sum of the expected future dividends (or cash flows), each discounted for the time and risk between now and the time the dividend is expected to be received − with the cost of capital r_S as the appropriate discount rate. The notion that the current stock "price equals the present value of expected future dividends" was first developed in 1938 by Williams and was then rediscovered by Gordon and Shapiro in 1956. [50]

1. Single-Stage DDM

If the dividend growth rate is constant, then we obtain the standard Gordon Growth model,[51] which can be shown to determine the cost of capital on security S as:

$$r_S = \frac{D_O \times (1 + g)}{P} + g \tag{8}$$

where g is the constant, periodical growth rate.

This equation says that the cost of capital equals the expected dividend yield (dividend divided by price) plus the (perpetual) expected future growth rate of dividends. As is readily seen from Equation (8) above, an implementation of the constant growth DDM requires a determination of the current stock price, current dividends, and the applicable growth rate.

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[49]   With the convention that D_i is zero for periods beyond the expected life of the asset.

[50]   See Brealey, Myers, and Allen (2011), p. 82.

[51]   Named after Myron J. Gordon, who published an early version of the model in "Dividends, Earnings and Stock Prices," *Review of Economics and Statistics*, Vol. 41, 1959, pp. 99-105.

2.   Multi-Stage DDM

If the assumption of constant growth is not considered reasonable for several years before settling down to a constant rate, variations of the general present value formula can be used instead. For example, if there is reason to believe that investors do *not* expect a steady growth rate forever, but rather have different growth rate forecasts in the near term (*e.g.,* over the next five or ten years) converging to a constant terminal growth, these forecasts can be used to specify the early dividends in Equation (7). Once the near-term dividends are specified, Equation (8) can be used to specify the share price value at the end of the near term (*e.g.,* at the end of five or ten years), and the resulting cost of capital can be determined using a numerical solver. A standard "multi-stage" DDM approach solves the following equation for r_S:

$$P = \frac{D_1}{\left(1+r_s\right)} + \frac{D_2}{\left(1+r_S\right)^2} + \ldots + \frac{D_T + P_{TERM}}{\left(1+r_S\right)^T} \tag{9}$$

The terminal price, P_{TERM}, is just the discounted value of all of the future dividends after constant growth is reached and T is the last of the periods in which a near-term dividend forecast is made. The implementation of the multi-stage growth model requires, in addition to a current price and current dividend, the selection of growth rates for each stage of the model and a determination of the length of each period.

More recent DDM implementations have focused on variations of the multi-stage model described above. For example, the U.S. Surface Transportation Board relies on a version of the multi-stage DDM that uses cash flow rather than dividends and specifies three growth rates – a near-term company-specific growth rate, an intermediate industry-specific growth rate and a long-term economy-wide growth rate.[52] The STB version is identical to the model developed by Morningstar / Ibbotson, Ibbotson's "three-stage" DDM, which is one of five models calculated for all U.S. SIC codes annually. In Ibbotson's version, dividends are replaced by cash flow (excluding extraordinary items) and the figure is normalized over a three-year period. The model then uses company-specific growth rates from analysts over the first five years, industry growth rates over the next five year and the GDP growth rate after year 10.

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[52]   See Surface Transportation Board, STB Ex Parte No. 664 (Sub-No. 1), "Use of a Multi-Stage Discounted Cash Flow Model in Determining the Railroad Industry's Cost of Capital," January 28, 2009. The Alberta Utilities Commission, Decision 2009-216 (¶271) also specifies a preference for the multi-stage model.

Another example of more recent multi-stage DDMs used is the version frequently estimated by *Brattle*, where company-specific growth rates are used for the first five years while the long-term GDP growth rate is used from year 10 onwards. In the in-between years (6-10), the model assumes that the growth rates converge linearly from the company-specific rates to the GDP growth rate. Similarly, Professor Myers' report suggests that in many industries it is important to look at the total cash flow that accrues to shareholders rather than on a per share basis, because stock buyback programs make the per share figures less reliable. In this model, the fundamental variable being determined is the market value (total price) of a company rather than the price per share, and instead of looking to dividends per share the model uses total cash flow to shareholders.[53]

3. DDM Implementation Issues

To implement the DDM it is necessary to specify one or more growth rates and to determine whether (i) dividends accurately reflect cash flow to shareholders, (ii) the horizon over which to apply each growth rate if using a multi-stage model, and (iii) the exact determination of the initial stock price. In most applications, the choice of growth rate is the most controversial part of the DDM implementation and the determination of the stock price is the least controversial.

4. Characteristics of the DDM

As for the other models, many of the strengths and weaknesses of the DDM depend on its implementation. However, assuming a reliable implementation, some strengths of the DDM are:

- Both the single-stage and the multi-stage DDM rely on forward-looking information and hence estimate a forward-looking cost of equity.

- The models are usually easily replicated and are therefore easy to audit.

Among the weaknesses of the DDM are the following:

- The DDM relies on growth forecasts, which frequently are available only for 2-5 years.

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[53] This revised method is explained in R. A. Brealey, S. C. Myers and F. Allen (2013), *Principles of Corporate Finance*, 11th Ed., McGraw-Hill Irwin, Ch. 16 (forthcoming).

- Because stock prices (and to a degree forecasted growth rates) change frequently, the model results often vary substantially over time.

Among the other issues to consider is the prevalence of stock buybacks, which means that dividends do not reflect all cash payments to shareholders. As mentioned above, some regulated entities have share buyback programs. In the pipeline industry, Spectra Energy, a U.S. based pipeline company, recently authorized share buybacks of $600 million for a little over 6% of its equity capital.[54]

Therefore, it is necessary to modify the model to take into account these cash transfers. In addition, for many companies, growth rates are only available on an infrequent basis, making the cost of equity estimates less forward-looking than ideal.

Both the single-stage and multi-stage DDM are frequently used in U.S. rate regulation to estimate the cost of equity. However, it is important to recognize that few U.S. regulators have a pre-specified methodology, but instead hear and review evidence from a variety of parties prior to issuing a decision on the cost of equity. Therefore, estimates from DDMs are only one of several pieces of evidence considered by most U.S. regulators. In addition, U.S. regulation was in place prior to the development of more market-based models such as the CAPM, and there is therefore a tradition to rely on the DDM.

5. Residual Income Model

One model that can be viewed as an extension of the multi-stage DDM is the residual income model, which relies on earnings or abnormal earnings instead of dividends. Broadly speaking, the model defines price as the sum of the book value of equity and the discounted present value of "abnormal" or "residual" earnings.[55] The model is a forward-looking methodology in that it generally uses analysts' forecasts to determine growth rates, although it uses historical earnings information to derive the current "residual income." The model is based on the so-called Ohlson-Juettner method, which like the multi-stage DDM allows growth rates to vary over time.

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[54] See Spectra Energy, Form 10-K, 2008 p. 31.

[55] For an early exposition, see J. Ohlson, "Earnings, book values, and dividends in equity valuation," *Contemporary Accounting Research* 11, pp. 661-687.

Abnormal earnings are typically forecast using earnings estimates for one or two years ahead. Assuming that abnormal earnings in the long run grow at the assumed long-run rate, the model allows for a high short-term earnings growth rate that gradually declines to the long-term level. Technically, the model is appealing because it provides a closed form solution to the cost of equity based on few inputs, so that it is simple to implement.[56]

The Residual Income Valuation (RIV) method has been debated substantially in the accounting literature in recent years. Variations on this model have been cited in recent Australian cases – for example, the "residual income model" proposed by the DBNGP in its most recent access arrangement.[57] The model was also proposed to the STB, albeit the STB instead adopted Ibbotson's three-stage DDM model based on cash flows rather than dividends.

In a recent paper by Nekrasov & Shroff (2009)[58] the authors propose a valuation methodology that applies risk measures based on economic fundamentals directly into the valuation model, aiming to assess the differences in valuation derived from the use of fundamentals-based risk adjustments instead of the commonly used asset pricing models (estimated using historical returns). Note that this paper does not specifically address valuation and cost of equity for the regulated entities.[59]

The authors use the RIV model to derive an accounting-based risk adjustment, which is equal to the covariance between a firm's ROE and economic factors. Accounting risk factors are identified and used to construct a measure of risk adjustment, then applied to calculate firm value. Two components of value are estimated separately: the risk-free present value (RFPV) and

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[56] The model was also submitted for consideration to the U.S. STB; P.S. Mohanram, *Determining an Appropriate Cost of Capital for Railroads*, submission to the Surface Transportation Board, September 2007.

[57] See *Draft Decision on Proposed Revisions to the Access Arrangement for the Dampier to Bunbury Natural Gas Pipeline*, paragraphs 458-467. Tristan Fitzgerald, Stephen Gray, Jason Hall and Ravi Jeyaraj, 2010 "Unconstrained estimates of the equity risk premium," Working paper, The University of Queensland, http://ssrn.com/abstract=1551748 ("Fitzgerald et al.").

[58] A. Nekrasov & P. Shroff, "Fundamentals-Based Risk Measurement in Valuation," *The Accounting Review* 84, 2009, pp. 1983-2011.

[59] See example or models submitted in regulatory settings; see Fitzgerald et al. and Partha Mohanram, "*Determining an Appropriate Cost of Capital for Railroads*," Submission to the U.S. Surface Transportation Board, September 2007.

the covariance risk adjustment. The RFPV is calculated using a forecast of earnings, book value of equity and the risk-free rate as inputs to the model, while the covariance risk adjustment is estimated by calculating betas on the different risk factors and corresponding factor risk premia.

The authors acknowledge that this methodology "may be more complex to implement than the returns-based cost of equity."[60] However, the authors conclude that the strong empirical performance of the one-factor accounting–beta model, combined with the need of few additional inputs for the estimation, justify its use in valuation applications.

6. Characteristics of the Residual Income Model

The pros and cons of the Residual Income Model are generally similar to those of the DDM model, but we note that the model considers earnings instead of dividends, so that if earnings and cash flows are reasonably consistent, this model better captures the totality of cash flow that accrues to shareholders.

F. OTHER MODELS, METHODS, MARKET DATA AND EVIDENCE

1. Risk Premium Approaches

Some regulators in North America use a simplified version of the CAPM, the so-called risk-premium approach, which collapses the beta and risk premium to one figure and adds this figure to an interest rate. The debt instrument is either government bonds or utility bonds. The risk premium approach calculates the cost of equity, r_S, as:

$$r_S = r_D + \text{estimated risk premium} \tag{10}$$

where r_D is the return on a selected debt instrument. There are many versions of this model depending on the choice of the debt instrument, r_D, and the estimation of the risk premium. It is important to note here that the risk premium approach, while a generalized form of the CAPM, does not have the same level of theoretical support as the standard CAPM. This is because the return on the selected debt instrument used is not necessarily equal to the risk-free rate, and the

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[60] *Ibid.* p. 1986.

estimated risk premium used is not explicitly based upon the product of the market beta and the MRP.

Equation (10) is frequently implemented using either a historical estimate of the risk premium, or a forward-looking or expected risk premium. The historical risk premium is commonly determined as the historical spread between equity and debt returns, so the primary choices for the analyst become which equity returns and debt instrument to use, as well as the period over which the spread (*i.e.*, the risk premium) is to be measured. It is not uncommon to see this model implemented using long-term government bonds or utility/corporate bonds to measure the cost of debt, while the equity investments used are typically either (a) realized accounting returns of regulated entities in the same industry, (b) realized stock returns of companies in the same industry, or (c) allowed returns on equity for the industry. In choosing a debt instrument to determine r_D, it is important that it be consistent with the debt instrument used to determine the risk premium. In other words, if a 10-year government bond is used to determine the historical risk premium, then r_D must also be measured using a 10-year government bond. The realized risk premium is highly dependent on the time period over which it is estimated, so that choice is also important.

The forward-looking model requires that the analyst determine a proper measure of the expected cost of debt and estimates the expected risk premium going forward, rather than relying on historical data. Determining the expected equity return is more difficult and requires reliance on an estimation technique. It is common to rely on DDM models to determine the risk premium in the forward-looking version of the model. One result originating from these analyses of historical or forward-looking risk-premium approaches is that empirically there is a negative relationship between the risk premium and the yield-to-maturity. Historically, a 1% increase in the yield-to-maturity of government bonds results in less than a 1% increase in the estimated (or realized) return on common equity.[61] The relationship between the return on equity and

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[61] For example, Roger A. Morin, "*New Regulatory Finance,*" Public Utilities Reports, Inc., 2006 pp. 128-129 summarizes several studies and found that the realized ROE changes approximately 50 basis points when government bond rates change 100 basis points. Regulatory agencies such as the Ontario Energy Board relied on this empirical finding as well as data submitted by experts in its recent hearing to update its annual change in the estimated cost of equity for Ontario utilities by less than the change in government bond rates.

(government or utility) bond yields is depicted in Figure 4 below. The figure is for illustrative purposes only and does not reflect an actual analysis of the relationship.
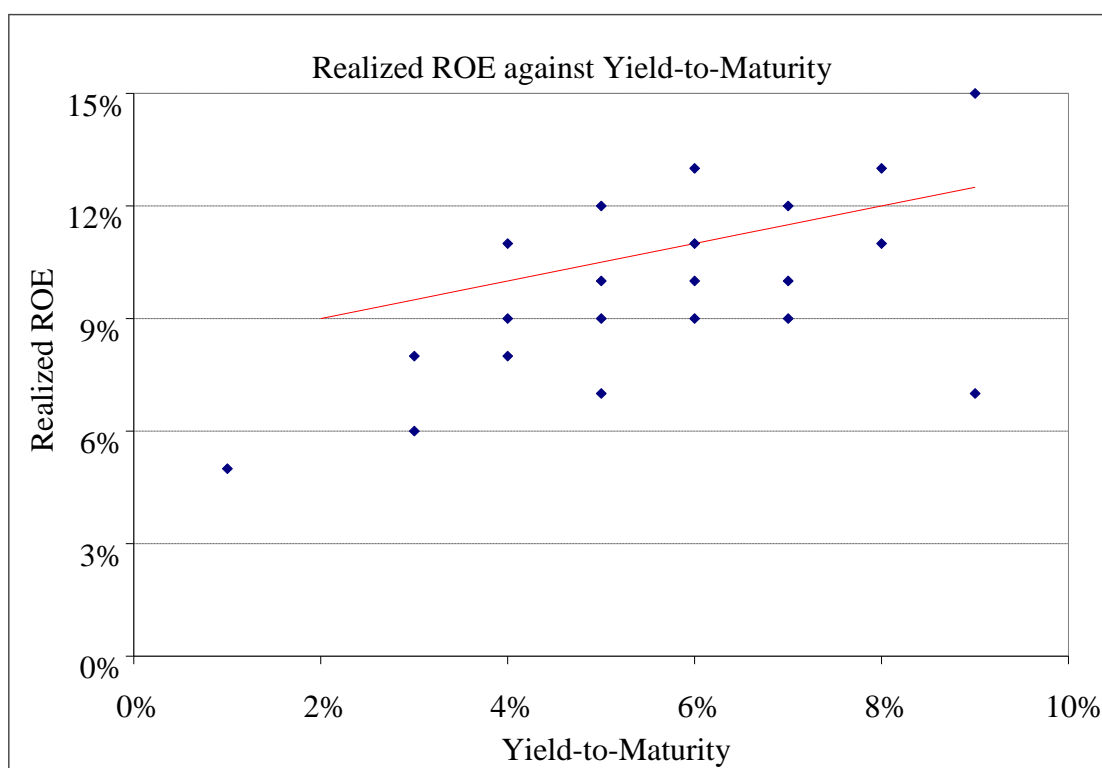
Figure 4

This is a reason why, for example, the Ontario Energy Board (OEB) took evidence from the risk premium approach into consideration when determining its baseline cost of equity in 2009.

2. Build-up Method

The build-up method estimates the return on an asset as the sum of a risk-free rate and one or more risk premia that represent the rewards an investor receives for taking on a specific risk:[62]

Cost of Equity = Risk-Free Rate + Market Risk Premium
+ Firm Size Premium + Industry Premium
+ potentially other factors

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[62] *Morningstar Ibbotson SBBI Valuation Edition 2012 Yearbook*, p. 27.

Each of the components of the build-up method is discussed in detail below:

- The Risk-Free Rate is calculated using either Treasury bills ('T-bills') or long-term government bonds.

- The Market Risk Premium reflects the compensation above the return on a risk-free asset that investors require for the additional market risk they bear by investing in a well-diversified market portfolio of risky assets. Ibbotson calculates this as the difference between the total expected return on the market portfolio and the risk-free rate.

- The Firm Size Premium may be included to account for the additional risk inherent in small company stocks. A firm size premium can either be adjusted or unadjusted for the effect that a small company stock's higher beta has on its excess return. To illustrate the magnitude of the size premium, Table 1 below shows the empirically observed size premium for U.S. companies as reported by Ibbotson Associates.

	Beta-Adjusted Size Premia (%)	Non-Beta-Adjusted Small Stock Premia (%)
Mid-Cap	1.1	1.9
Low-Cap	1.9	3.4
Micro-Cap	3.9	6.3
Small Company Stocks	3.1	4.7

Table 1: Ibbotson Associates' Size Premia on a Beta-Adjusted versus Non-Beta-Adjusted Basis, 1926-2011[63]

- An Industry Premium can be determined based on the characteristics of the regulated entity's industry. Research has produced no consensus on this figure and Ibbotson notes that it is important to avoid double-counting industry risk by using other beta-adjusted (hence industry dependent) risk premia (positive or negative) and at the same time adding an industry premium.

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[63] *Morningstar Ibbotson SBBI Valuation Edition 2012 Yearbook*, p. 27.

In addition to the factors discussed above, some argue for the inclusion of minority discount premia, control premia, key person discount, *etc.* However, these additional premia (positive or negative) are very difficult to measure and we know of no regulator that has included such additional factors. The New Mexico Public Regulation Commission in the U.S. has in the past used the build-up method as one of its methods to estimate the cost of equity.

3. Comparable Earnings

The comparable earnings method requires the analyst to go through three steps. First, a group of *unregulated* companies is required because the realized accounting rate of return of a regulated company depends on its allowed return. Using regulated companies to estimate the comparable earnings cost of capital would be circular, *i.e.*, the allowed rate of return is used to determine the allowed rate of return. However, the use of unregulated companies requires careful consideration of the risk characteristics of the companies and the comparability to those of the target utility.

Second, a time period over which to estimate the return on equity must be selected. Because a company's achieved earnings fluctuate from year to year and depend substantially on both company-specific and economy-wide factors, it is necessary to include companies from several industries, averaged over several periods.

Third, because the comparable companies are unregulated entities, it is necessary to adjust for any risk differences between the sample companies and the target company. There are many ways to adjust for risk differences, so the following is a simplified description of some common approaches rather than an exhaustive review. Analysts often collect information on the comparable companies' and the target company's bond ratings, asset betas, DDM estimates of the cost of equity, and other measurable risk factors. In many instances, this information is also collected for a sample of regulated companies in the same industry as the target company. If the sample companies are found to be consistently more (less) risky than the target company and its industry peers, then an adjustment is made to the required return on equity. This can sometimes be done formally. For example, if the sample companies' DDM estimates of cost of equity are consistently 25 basis points higher (lower) than the DDM estimates for the target company (or industry peers), then a downward (upward) adjustment of 25 basis points is made. For other

measures, it is more difficult to determine the exact adjustment, so it is usually made based on the analyst's experience. For example, does a two notch difference in bond rating require a specific upward or downward adjustment? Thus, while the differences are relatively easy to measure, the adjustment for such differences requires subjective judgment.

A major issue is whether realized book returns are a good proxy for the returns that investors expect going forward. From a statistical perspective, the realized accounting return on book equity for any given period is the realization of a single outcome of a distribution, whereas the expected return represents the probability-weighted average of all possible outcomes of the distribution. These two figures can differ substantially. In addition, there are practical problems with the implementation of this model because financial reporting occurs with a lag, which during times of change can mean that the results are out of date.

4. Market-to-Book and Earnings Multiples

In some regulatory decisions on the cost of capital, regulators have sought to "cross check" a proposed cost of capital estimate by examining the market value of the firms they regulate relative to the value of the regulatory asset base (RAB). The theory behind this approach would be that the only capital on which the regulated firm is earning a return (at the regulator-determined cost of capital) is the RAB. Therefore, if the market value of the firm's returns is greater than the RAB, the belief is that it is a signal that investors are discounting future returns at a lower discount rate than the regulator's cost of capital determination — or, in other words, the regulator's cost of capital is "too high".

This kind of cross check approach was cited by the Australian Energy Regulator in its June 2011 determination on Envestra.[64] In that decision, the AER considered two kinds of evidence: premiums paid in takeover transactions relative to the value of the RAB, and market values (based on share prices) relative to RAB.

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[64] *Final Decision - Envestra Ltd Access arrangement proposal for the Qld gas network*, AER (June 2011), p. 35-37.

a) *Takeover premiums*

The AER reviewed premiums paid in takeover transactions, where the premium was assessed as the sale price relative to the value of the underlying RAB. Premiums were in the range of 20% to 120%. The AER considered that these premiums were too large to be explained by factors such as expected synergies, and instead considered this as evidence that the cost of capital determined by regulators has been at least as high and likely higher, than the actual cost of capital faced by the businesses.

However, there are conceptual problems with this approach so that it has no value as a cross check on a regulator's cost of capital determination. First, the reliance on the approach implicitly assumes that (i) the company to which it is applied consists entirely of regulated businesses and (ii) that the regulator's cost of capital determination is the only factor impacting the market value of the company. In reality the cost of equity is only one component of a broader determination on what the firm's regulated rates should be. Thus, even if it were possible to estimate the impact of the regulator's decision on the market value of the firm, this impact would be associated with the overall decision, not with any one specific component (like the cost of capital). The market value of a regulated firm can be thought of as the expected future cash flows (from providing services at regulated rates), discounted at the firm's actual cost of capital. However, the regulator's cost of capital determination is only one of many factors which determine expected future cash flows, particularly where price determinations are forward-looking (as in Australia):

- If investors expect the firm to "beat" regulator assumptions on any of operating costs, capital costs, or revenue growth, expected future cash flows would be larger than the RAB in net present value terms, even if the discount rate is equal to the regulator-determined cost of capital.

- Investor expectations, which are implicit within the firm's market value, encompass expected cash flows beyond the end of the current price control period.

- Expected future cash flows are also affected by firm-specific factors such as idiosyncratic volatility, which would not be captured in the discount rate.

In addition, there are likely to be other more practical difficulties: for example, many regulated firms have at least some unregulated activities. These activities are valued by investors but are not part of the RAB or the regulator's cost of capital decision.

b) Trading premiums

The AER also considered premiums measured on the basis of market value of listed firms (from share prices) relative to RAB. The AER estimated market-to-RAB trading multiples for four firms (including Envestra).[65] The trading multiples were in the range of 1.21 to 1.81.

The AER stated that these premiums were too high to be the result of factors such as expected synergies, and instead considered this as evidence that the cost of capital determined by regulators has been at least as high and likely higher, than the actual cost of capital faced by the businesses. However, the same difficulties described above for takeover premiums also apply to the consideration of trading premiums. In addition to the takeover premiums difficulties, the use of trading premiums suffers from bias in circumstances where the market is very volatile, where day-to-day changes reflect investor reactions to news such as the collapse of Lehman Brothers in September 2008, the ongoing European debt crisis, or industry factors such as cap and trade initiatives, etc. Therefore, trading premiums also have no value as a cross check on the regulator's cost of capital determination.

5. Other Evidence

Other evidence is a very broad category that does not readily lend itself to a short introduction by method. However, expert evidence can be highly valuable if of high quality, so it will be necessary to use judgment and consider how the expert arrived at his or her recommendations. Similarly, academic research may provide insights into the cost of equity, but bear in mind that most academic research focuses on finding or explaining "interesting facts" and often considers all companies and industries for which data are available. Because a result pertains to the market

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[65] The four firms were SP Ausnet, Spark, Duet and Envestra (*Ibid.*, Table 5.5).

as a whole, it does not necessarily pertain to a specific industry, which may have unique characteristics.

Other types of evidence that are sometimes considered are equity analysts' reports on a specific company, an industry, or a market. When such evidence is reviewed, it is important to consider the purpose for which the evidence was produced. For example, equity analysts often produce research documents aimed at stock-buying investors and only rarely are concerned with the cost of equity over, for example, a regulatory period. Instead, equity analysts attempt to determine the current (or future) stock price as the discounted sum of future cash flows with the discount rate being the weighted average sum of the cost of debt and equity; *i.e.*, the focus is not on what the best estimate of the cost of equity is – it is merely one of many inputs to determining the stock price. In addition, because a lower cost of equity increases the estimated stock price, equity analysts have an incentive to, if anything, bias the cost of equity estimates downward.

6. Characteristics of Other Methods, Models, Market Data and Evidence

The methods, models, market data and other evidence in this section differ, so the advantages and disadvantages listed below are method-specific:

- The risk premium model is simple and data for its implementation are readily available.

- If the benchmark interest rate is a utility or corporate bond index, then the risk premium model tends to provide relatively stable results over time and is less impacted by monetary policy or country-specific risks than the CAPM.

- The build-up method recognizes size effects and potentially other risks.

- The comparable earnings method's strength is that it incorporates information from non-regulated entities.

Among the weaknesses of the methods we note the following:

- None of the methods are founded in economic or finance theory.

- The risk premium approach does not consider systematic risk specifically and does not allow for company-specific information to be considered.

- The build-up method generally does not consider systematic risks and treats size effects the same across industries.

- The comparable earnings model relies on historic accounting information, which may not be consistent with investor expectations. Also, the historic accounting information may reflect accounting choices rather than economic fundamentals and may be subject to significant variability over time.

As for other evidence such as expert reports and investment reports, the merits of the derived estimates are highly dependent upon the quality of the reports and the purpose for which the estimates were derived. We caution against placing weights on estimates where the purpose for their derivation is not known, and against placing substantial weight on estimates that were derived for purposes other than to provide an independent assessment of the cost of equity. For example, estimates derived for accounting purposes, stock recommendations, *etc.* may not be suitable for other uses.

This section has summarized the major models, methods and evidence that are currently used and considered by regulators and practitioners. The models described above are not intended to comprise an exhaustive list of all possible methods and evidence that could be relied upon in determining the cost of equity capital. Indeed, as the practice of finance continues to evolve, further relevant evidence may still be found, and certain models may become outdated or less relevant.

IV.   USING THE METHODS

In this section, we first discuss implementation issues for estimating the cost of capital and summarize the key characteristics of the models described above in *Section III*. We then address the issue of how and when to use the models to determine an appropriate regulatory return on equity, or range for the regulatory return on equity for the industry or benchmark, based on the views of academic, practitioners and regulators. Finally, we discuss how to position a target entity relative to a sample of companies.

A.   IMPLEMENTATION ISSUES

Regardless of the cost of equity estimation method that is used to estimate the cost of capital, there are some key elements of the cost of capital estimation process that must be addressed. This section discusses some of the important issues.

Most analysts rely on a "comparable sample" to determine the cost of equity for the target entity, so it becomes important to determine what is meant by comparable.[66] Although the selection of comparable companies is method and context-specific, it is generally viewed as ideal to have sample companies with business risk similar to that of the target company. Similar business risk generally implies selecting companies in the same line of business. Most researchers and practitioners rely on additional criteria to exclude sample companies that have the potential to bias the cost of capital estimation methodologies. For screening, it is preferable to rely on objective information from publicly available data sources; however, the determination of exactly which criteria to use is subject to the constraint that the sample be "large enough." This, in turn, requires a determination of which criteria are the most important from the many possible criteria that could be considered. Among the criteria typically employed are combinations of the following:

- Include companies with similar business risks (*e.g.,* companies in the same or similar industries);

- Exclude companies that face financial distress;

- Exclude companies that are or have recently been involved in substantial merger and acquisition activity;

- Exclude companies with unique circumstances that may bias the cost of capital estimation (*e.g.,* restatements of financial statements); and

- Exclude companies with insufficient data.

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[66] A comparable sample can be used to assess the cost of capital for the target entity by (i) estimating the individual companies' cost of capital and placing the target company's cost of capital in relation to the sample using the average, median, range, or other measure to assess the cost of capital or (ii) using a portfolio approach, where the cost of capital for the portfolio of companies (rather than individual companies) is estimated to assess the cost of capital for the target entity.

There is, however, controversy about how to implement the criteria above. Each element of the sample selection criteria requires some judgment. For example, what size sample is "large enough"? Should the sample include both Australian and foreign companies?[67] How is financial distress measured? How is "substantial merger and acquisition" activity to be defined? The selection criteria are interrelated, because selection of the sample based upon one criterion may immediately reduce the potential sample to a small number of companies. The sample selection process is, therefore, a balancing act between selecting a sample that is "more comparable" and one that is "too small."

Second, decision makers must decide how the components of the cost of capital will be determined. For example, it is possible to estimate (a) the cost of debt, the cost of equity and the capital structure, each separately or (b) an overall cost of capital or (c) a combination of these. Another component of the cost of capital is the allowance for income taxes, which we ignore in this report. Finally, because the dollar amount that accrues to investors in a regulated entity ultimately depends on not only the allowed cost of equity and the size of the rate base but also on the relative share of equity and debt in the capital structure, it is important to consider the overall impact of these capital structure decisions on the individual components. Specifically, it is important to note that cost of equity estimation models provide estimates that reflect both the underlying business risk of the assets but also the financial risk inherent in how those assets have been financed.

B. SUMMARY CHARACTERISTICS OF THE MODELS

Before we discuss how to use the various models and other information that may be available to a decision maker, we summarize in Table 2 below the key characteristics of the discussed models in the form of their economic underpinnings, any potential empirical bias, sensitivity to economic or industry factors, and whether the models are forward or backward-looking.

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[67] For example, several Canadian regulators have used beta estimates from U.S. companies. See, for example, the National Energy Board's RH-1-2008 decision p. 67 and Ontario Energy Board's EB-2009-0084 decision, pp. 22-23.

Table 2: Characteristics of Cost of Equity Methods

Cost of Capital Methods	Evaluation Criteria			
	Economic Underpinnings	Bias	Impact of Market Conditions	Forward or Backward-Looking
Sharpe-Lintner CAPM	An equilibrium model: Under no arbitrage and in a mean-variance-optimizing world, the expected cost of equity is a function of the risk-free rate, systematic risk (beta) and the expected MRP. Transparent and sensitive to market performance and risk-free rates. Empirical support for explaining cross-sectional returns of average-beta stocks, but failure for low-beta/high-beta/small/high book-to-market firms.	Empirical evidence that CAPM under-estimates the expected return for low-beta stocks. A Portfolio approach to estimate betas provides more consistent results. MRP estimation controversial with some historical measures potentially biased.	Sensitive to monetary policy. Market uncertainty and economic turmoil likely to affect the expected MRP.	Beta estimates are backward-looking. Historical MRP is backward-looking, but forecast MRP (e.g., DDM) are forward-looking.
ECAPM	Same as above, but captures the empirical observation that the SML predicted by CAPM is too steep. Tested in the U.S., but not extensively outside the U.S. or in recent market conditions.	Corrects for the empirical bias induced by the flatter-than-predicted-by-CAPM Security Market Line.	Same as above.	Same as above.

(Table 2 ctd.) Characteristics of Cost of Equity Methods

Cost of Capital Methods	Evaluation Criteria			
	Economic Underpinnings	Bias	Impact of Market Conditions	Forward or Backward-Looking
Consumption-Based CAPM	Generalization of the Sharpe-Lintner CAPM that relates the risk premium on the investment to the covariance between the asset return and the intertemporal marginal rate of substitution of the decision maker. The expected conditional risk premium on an asset is related to its predicted conditional volatility. Requires more data and more refined estimation techniques than the Sharpe-Lintner CAPM. Lack of empirical support for most commonly used version (where covariance factor is aggregate consumption growth), but more support for versions with market frictions/time-varying risk aversion.	Allows for expected risk premium to vary with asset and investor characteristics, including conditional volatility and covariance with consumption growth; (considers more factors than the Sharpe-Lintner CAPM). Potentially mitigates empirical biases.	Empirical results appear stable and consistent over time (i.e. more robust to market conditions than standard CAPM).	Models forward-looking equity risk premia based on predicted conditional volatility. More forward-looking than Sharpe-Lintner CAPM.

(Table 2 ctd.) Characteristics of Cost of Equity Methods

		Evaluation Criteria		
Cost of Capital Methods	**Economic Underpinnings**	**Bias**	**Impact of Market Conditions**	**Forward or Backward-Looking**
Fama-French Model	Corrects for empirical biases of Sharpe-Lintner CAPM by adding 2 explanatory risk factors (size and book-to-market). Empirical support for explaining cross-sectional returns of size- and book-to-market-sorted portfolios. Weak empirical support for explaining returns of other portfolios and for out-of-sample predictive power. Fama-French factors (SMB and HML) are empirically motivated.[1]	Captures empirical observation that SML predicted by CAPM is too steep. Adds cross-sectional explanatory power to the standard CAPM.	Sensitive to monetary policy. Estimates of the 3 factor risk premia vary substantially over time and more so when the 3 factors interact.	Betas and factor risk premia estimates are backward-looking.
APT Model	Equilibrium multifactor model which holds under competitive markets, factor structure for asset returns, and absence of arbitrage in large economies. Corrects for empirical biases of Sharpe-Lintner CAPM by adding explanatory risk factors. APT factors are theoretically motivated. Model implemented empirically as intertemporal CAPM (see above for empirical issues).	Same as above.	Sensitive to market uncertainty and economic turmoil via market-related factor. Estimates of factor risk premia can vary substantially over time.	Forward-looking model in theory, but betas and factor risk premia are backward-looking if estimated using historical data.

1) See Fama and French (1993), Kothari, Shanken, and Sloan (1995), Black (1993), MacKinlay (1995), and Lakonishok, Shleifer, and Vishny (1994) for additional detail.

(Table 2 ctd.) Characteristics of Cost of Equity Methods

Cost of Capital Methods	Evaluation Criteria			
	Economic Underpinnings	Bias	Impact of Market Conditions	Forward or Backward-Looking
Risk Premium Model	Simplified version of the CAPM, which collapses the beta and market risk premium to one figure and adds this figure to an interest rate. Based on empirical estimation. Does not capture systematic risk or company-specific information.	The relied upon interest rate may be biased due to, e.g., monetary policy. Does not account for changes in the risk premium.	Using a utility or corporate bond index as the benchmark, the risk premium model tends to provide relatively stable results. Reliance on government interest rates makes the model more sensitive to monetary policy. Inflation leads to bias in the risk premium model, because the historical data underlying the estimate of the risk premium may not be consistent with the current level of inflation.	Mostly backward-looking due to reliance on a historic spread of realized equity returns over debt returns to measure the risk premium, and thus does not capture expected changes in the economy.
Single-Stage DDM	Determines today's stock price as the sum of the discounted cash flows that are expected to accrue to shareholders going forward. Assumes that dividends (or cash flows) grow at a constant rate forever. Lack of empirical support for constant dividends/earnings growth rates in perpetuity.	Sensitive to bias in analyst forecasts of earnings growth rates which at best reflect 5 years. Less of an issue for utilities than most other industries. Sensitive to the exact implementation as dividends may not reflect all cash flow if the company engages in share repurchases or borrows to fund dividends. Does not take real options into account and will underestimate the cost of equity for companies with substantial real options.	Model requires the constant-growth assumption. Estimates are sensitive to changes in stock prices and forecasted growth rates, which is especially an issue if the industry is in transition. Stock prices are influenced by the information available to investors. Information about financial distress or merger and acquisition activities may overwhelm fundamental information about growth.	Relies on forecasted (i.e. forward-looking) growth rates and current stock prices. Hence estimates are forward-looking.

(Table 2 ctd.) Characteristics of Cost of Equity Methods

Cost of Capital Methods	Economic Underpinnings	Bias	Impact of Market Conditions	Forward or Backward-Looking
			Evaluation Criteria	
Multi-Stage DDM	Extension of single-stage DDM which allows for different growth forecasts over time. Stronger empirical support than constant growth version.	Same as above, but less sensitive to any bias in analyst forecasts than the single-stage DDM, as growth rates usually converge to the GDP growth rate over time.	Requires a series of growth rates; commonly near-term, interim, and for the very long-term. Sensitivity to growth rates is moderated. Similarly to single-stage DDM, less applicable to companies in financial distress or engaged in merger or acquisition activities.	Same as above.
Residual Income Method	Version of the multi-stage DDM that values abnormal or unforeseen earnings instead of dividends. Abnormal earnings are forecast using earnings estimates for one or two years ahead. Allows growth rates to vary over time. Abnormal earnings are based on empirical estimates.	Considers earnings instead of dividends or cash, so if earnings reflect expectations better than dividends or cash, it reduces bias.	Same as above. Performance relative to multi-stage DDM depends on implementation of each.	Same as above.

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(Table 2 ctd.) Characteristics of Cost of Equity Methods

Cost of Capital Methods	Evaluation Criteria			
	Economic Underpinnings	Bias	Impact of Market Conditions	Forward or Backward-Looking
Build-up Method	Estimates the return on an asset as the sum of a risk-free rate and several risk premia that measure risks associated with size, industry, etc. Based on empirical estimation.	Recognizes size effects and other industry or company-specific risks. Exposed to same potential biases as standard CAPM and Fama-French models.	Exposed to same market uncertainties as standard CAPM and Fama-French models.	If risk factors and factor loadings are estimated from historical data, then the model is backward-looking.
Comparable Earnings	The model calculates the realized accounting rate of return on book equity of comparable (usually non-regulated) companies. Based on empirical estimation. Uses accounting returns rather than market data.	Selecting a sample of non-regulated comparable companies may lead to bias. Accounting changes can produce changes in model estimates without any change in the underlying cost of capital. The choice of estimation period may bias the accounting return on equity as accounting returns vary with economy, industry and company-specific factors.	Realized accounting returns are sensitive to economic, industry and company-specific events as well as to changes in accounting rules.	Uses backward-looking realized accounting rates of return, hence backward-looking. Can be difficult to find a time period that accurately reflects the expected horizon of the regulated entity.

C. HOW TO USE THE MODELS AND OTHER INFORMATION

In this section we discuss how academics, practitioners and regulators think models should be used and how they have been used. The section also discusses the impact of economic conditions, industry factors and company-specific issues on the choice of models. The weight assigned to each model naturally depends on the key characteristics of the cost of equity estimation models described above. Finally, the section discusses how certain regulators have decided to use the models in specific economic environments.

1. Views of Academics, Practitioners and Regulators

Academics, practitioners and regulators have all acknowledged that there is no one way to determine the cost of equity. In the academic literature, several prominent researchers have commented that the use of more than one method is important. For example, Professor Myers of the Massachusetts Institute of Technology commented:

> Use more than one model when you can. Because estimating the opportunity cost of capital is difficult, only a fool throws away useful information. That means you should not use any one model or measure mechanically or exclusively. Beta is helpful as one tool in a kit, to be used in parallel with DCF models or other techniques for interpreting capital market data.[68]

Professors Berk and DeMarzo of Stanford University in their corporate finance textbook comment on the use of the CAPM, DDM, and other models by practitioners, and state:

> In short, there is no clear answer to the question of which technique is used to measure risk in practice — it very much depends on the organization and the sector. It is not difficult to see why there is so little consensus in practice about which technique to use. All the techniques we covered are imprecise. Financial economics has not yet reached the point where we can provide a theory of expected returns that gives a precise estimate of the cost of capital. Consider, too, that all techniques are not equally simple to implement. Because the tradeoff between simplicity and precision varies across sectors, practitioners apply the technique that best suit their particular circumstances.[69]

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[68] Stewart C. Myers, "On the Use of Modern Portfolio Theory in Public Utility Rate Cases: Comment," *Financial Management*, Autumn 1978.

[69] Jonathan Berk and Peter DeMarzo, *Corporate Finance: The Core*, 2009, (Berk & DeMarzo 2009) p. 420.

Looking to practitioners' views, the widely used text, *Ibbotson Cost of Capital Yearbook*,[70] reports results on the cost of equity (and associated weighted average cost of capital) by SIC code in the U.S. and other countries. In doing so, the yearbook reports the estimated cost of equity using five estimation methods: Sharpe-Lintner CAPM, CAPM plus/minus a size premium, Fama-French 3-Factor model, Single-Stage DDM, and 3-Stage DDM. The data source does not provide specifics on how to use the data but states that:

> [r]eaders can select cost of equity from five different models explored in this book. Given the size of the database being analyzed, there will clearly be instances where certain cost of equity models will fail to produce useable numbers. When NMF is displayed in a cost of equity column, it indicates that the model is producing unreasonable numbers, and greater emphasis should be placed on other models.[71]

Similarly, Roger A. Morin, in the context of U.S. regulation, mentions the use of the CAPM, DDM, risk premium models, and the comparable earnings method, concluding:

> No one individual method provides the necessary level of precision for determining a fair return, but each method provides useful evidence to facilitate the exercise of an informed judgment. Reliance on any single method or preset formula is inappropriate when dealing with investor expectations because of possible measurement difficulties and vagaries in individual companies' market data.[72]

Looking to regulators, the U.S. Surface Transportation Board (STB) undertook a review of its cost of equity estimation methodology in 2007-09 in two rounds, focused on the CAPM and DDM respectively. The STB's review resulted in two decisions with detailed instructions on how to estimate the cost of capital for the railway industry.[73]

In connection with this review, the STB noted:

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[70] The most recent version is Morningstar, *Ibbotson Cost of Capital 2012 Yearbook* (Ibbotson 2012).

[71] Ibbotson 2012, p. 6. The text views cost of equity estimates below the risk-free rate and above 50 percent as being not meaningful.

[72] Roger A. Morin, *New Regulatory Finance*, Public Utilities Reports, Inc., 2006, (Morin 2006) p. 428.

[73] Surface Transportation Board, STB Ex Parte No. 664, "Methodology to be Employed in Determining the Railroad Industry's Cost of Capital," January 17, 2008 (STB 2008) and STB Ex Parte No. 664 (Sub-No. 1), "Use of a Multi-Stage Discounted Cash Flow Model in Determining the Railroad Industry's Cost of Capital," January 28, 2009 (STB 2009).

While CAPM is a widely accepted tool for estimating the cost of equity, it has certain strengths and weaknesses, and it may be complemented by a DCF model. In theory, both approaches seek to estimate the true cost of equity for a firm, and if applied correctly should produce the same expected result. The two approaches simply take different paths towards the same objective. **Therefore, by taking an average of the results from the two approaches, we might be able to obtain a more reliable, less volatile, and ultimately superior estimate than by relying on either model standing alone** [emphasis added].[74]

In arriving at this conclusion, the STB took notice of comments from the Federal Reserve that "multiple models will improve estimation techniques when each model provides new information,"[75] and also stated that there is "robust economic literature confirming that, in many cases, combining forecasts from different models is more accurate than relying on a single model."[76]

Similarly, the Ontario Energy Board (OEB) reviewed its cost of capital estimation methodology in 2009 following a year-long process. For context, the OEB does not focus on the cost of equity, but instead determines the premium over the risk-free rate that rate-regulated utilities are allowed. Regarding the methods used to determine the so-called Equity Risk Premium (ERP), the OEB concluded:

> **the use of multiple tests to directly and indirectly estimate the ERP is a superior approach to informing its judgment than reliance on a single methodology**.[77]

Additional examples of regulators who have relied upon multiple cost of equity estimation models and/or judgment based on a range of evidence are discussed in the section below.

To sum up, as clearly illustrated above, many academics, practitioners and regulators find that it is preferable to use more than one estimation method to determine the cost of equity. We agree that it is important to use more than one estimation method and stress that in determining how to

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[74]   STB 2008, p. 2.

[75]   STB 2009, p. 15.

[76]   STB 2009, p. 15.

[77]   Ontario Energy Board, "EB-2009-0084, Report of the Board on the Cost of Capital for Ontario's Regulated Utilities," Issued December 11, 2009, p. 36 (emphasis in the original).

weigh the estimation results, it is important to consider the degree to which the information from the methods overlaps versus providing additional information, the economic and financial environment that gave rise to the estimates, and the context in which they are being used.

2. Regulatory Practice in using Multiple Models

a) The U.S.

In the U.S., rates for rate-regulated entities are determined by several federal entities as well as regulators in each of the fifty states and the District of Columbia. Federal regulators tend to have well-specified methods to determine the cost of equity although they review all the information put to them. However, state regulators typically do not specify one single method and commonly have evidence from several estimation methods and parties in front of them before issuing a decision on the allowed cost of equity. In most cases the state regulator does not specify which weight was assigned to each method or other evidence. An exception is the determination of the cost of equity in Mississippi Power's Performance Evaluation Plan (PEP), where the Mississippi Public Service Commission annually updated the cost of equity for the company using a combination of the CAPM, ECAPM, risk positioning, and the DDM. In this specific circumstance, the weights assigned to each method are predetermined.[78] Some other examples of U.S. regulators' thought processes are provided below.

Surface Transportation Board

The STB used the constant growth model to track the cost of equity for U.S. railroads for a number of years. However, by 2005 the largest railroads were expanding rapidly and profitability was increasing. Security analysts were forecasting "long-run" earnings growth for some railroads at 15% per year. Such growth could not be sustained, so the constant growth model overstated the true cost of equity by a wide margin. The STB therefore initiated a cost of capital proceeding to consider how to change the determination of the cost of equity. After hearing evidence from academics and practitioners, the STB found that:

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[78] http://www.psc.state.ms.us/.

if our exploration of this issue has revealed nothing else, it has shown that there is no single simple or correct way to estimate the cost of equity for the railroad industry, and countless reasonable options are available.[79]

As a result of its deliberations the STB eventually settled on a blend of the CAPM and a multi-stage DDM.[80]

Georgia

The following example pertaining to Georgia Power, an integrated electric utility, illustrates a common approach in U.S. state regulation.

Georgia Power is regulated by the Georgia Public Service Commission (Georgia PSC), which has no pre-set method to determine the cost of equity. In Georgia Power's 2010 rate case, an expert for Georgia Power as well as for the Georgia PSC submitted evidence on the cost of equity for the company. The company's expert estimated the cost of equity using the Sharpe-Lintner CAPM, a single-stage DDM, and a risk premium approach, and recommended a return on equity of 11.0 to 11.2%. The PSC staff expert estimated the cost of equity using the Sharpe-Lintner CAPM, a sustainable growth DDM and also a comparable earnings model for a recommendation of 9.50 to 10.75%. The Georgia PSC approved a settlement including a cost of equity of 11.15%, but did not specify how it was arrived at.[81]

b) Canada

Until the early 1990s, Canadian regulators, much like U.S. state regulators, heard evidence on a multitude of methods and from various experts before arriving at a decision on the allowed cost of equity. However, starting in British Columbia in 1994, the British Columbia Utilities Commission in the first generic cost of capital proceeding in Canada established a benchmark ROE and a formulaic approach to updating the allowed ROE annually.[82] Shortly thereafter, other Canadian regulators followed suit and similarly established a benchmark ROE and an

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[79] *U.S.* Surface *Transportation Board, Ex Parte 664 (Sub-No. 1)*, issued January 28, 2009, p. 15.

[80] *Ibid.*

[81] Direct Testimony of J.H. Vande Weide in Docket No. 31958; Direct Testimony of D. Parcell in Docket No. 31958, and Settlement Agreement in Docket No. 31958.

[82] BCUC Decision in the Matter of Return on Common Equity BC Gas Utility Ltd., Pacific Northern Gas Ltd., West Kootenay Power Ltd., June 10, 1994 (BCUC 1994 Decision), pp. 39-40.

annual updating formula. These formulae were linked to the change or forecasted change in government bond yields.

While the formula used to update the allowed ROE annually was mechanical, the methods used to estimate the benchmark ROE varied across jurisdictions, and in many jurisdictions, the regulator looked to more than one estimation method.[83]

As the yield on government bonds declined, so did the allowed cost of equity, and as the financial crisis of 2008 impacted financial markets, regulators in Canada abandoned or modified the formula or the relied-upon benchmark. As was the case for the originally developed benchmark, the regulators heard evidence on multiple methods from several experts and implicitly or explicitly weighted these methods to arrive at a new or modified cost of equity methodology.[84] Some examples of this regulatory approach in Canada are provided below.

British Columbia

British Columbia Utilities Commission's (BCUC) views on how to determine the appropriate cost of equity capital have evolved over time. In the BCUC 1994 Decision,[85] the BCUC "placed primary reliance on the various risk premium tests presented" whereas the "comparable earnings and DCF test results have been used primarily as a check upon reasonableness."[86] However, in the BCUC 2006 Decision, the BCUC assigned weight to the DCF model and found the comparable earnings methodology useful.[87] The BCUC 2006 Decision did not state how much weight it assigned to each model it considered. The BCUC's views evolved as the various

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[83] For example, the BCUC 1994 Decision at p. 17 indicated that while primary reliance should be placed on risk premium tests, comparable earnings and the DDM should be used as checks.

[84] For example, the National Energy Board abandoned the formulaic approach, the Alberta Utilities Board modified the benchmark, and the Ontario Energy Board modified both the benchmark and the formula. Both the Alberta Utilities Board and the Ontario Energy Board used several cost of equity estimation methods to arrive at their revised benchmark. The British Columbia Utilities Commission is in the midst of a generic cost of capital proceeding that will determine the approach going forward.

[85] BCUC Decision in the Matter of Return on Common Equity BC Gas Utility Ltd., Pacific Northern Gas Ltd., West Kootenay Power Ltd., June 10, 1994 (BCUC 1994 Decision).

[86] BCUC 1994 Decision, p. 17.

[87] BCUC In the Matter of Terasen Gas Inc. *et al.* Return on Equity and Capital Structure Decision, December 16, 2009 (BCUC 2009 Decision), pp. 44-45.

models arrived at more or less plausible results. For example, in its 2009 decision, the BCUC found:

> The Commission Panel agrees that a single variable is unlikely to capture the many causes of changes in ROE and that in particular the recent flight to quality has driven down the yield on long-term Canada bonds, while the cost of risk has been priced upwards.[88]

Having acknowledged the influence of the current economic environment, the BCUC in 2009 gave the most weight to the DDM, less weight to the Equity Risk Premium method and CAPM, and a low weight to the comparable earnings model. While the BCUC acknowledged giving weight to the DDM, ERP, CAPM and comparable earnings method, it did not specify the exact weights used.[89] The BCUC is currently undertaking a review of its cost of capital estimation methodology.

Ontario Energy Board

The Ontario Energy Board ("OEB") regulates electric and gas utilities in Ontario and sets rates for electric and natural gas distribution and transmission. The OEB also regulates other aspects of the electric and natural gas sector, but it does not regulate competitive electric or gas supply. In addition to determining the allowed cost of capital, the OEB also determines a deemed (allowed) capital structure for the utilities it regulates, and the allowed cost of equity is applied to the deemed equity portion of the allowed rate base, which is based on historical cost.

The OEB reviewed its approach to determining the cost of capital for Ontario utilities and in December 2009 issued a report on its estimation procedures going forward.[90] Prior to the review, the OEB relied on a formula-based approach using a version of the risk premium approach, or Equity Risk Premium (ERP) method to determine the return on common equity. Although a number of concerns were raised with this approach, the OEB decided to continue

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[88]   BCUC 2009 Decision, p. 73.

[89]   BCUC 2009 Decision, p. 45.

[90]   Ontario Energy Board, "EB-2009-0084, Report of the Board on the Cost of Capital for Ontario's Regulated Utilities," Issued December 11, 2009 (OEB Report 2009).

relying on a formula-based methodology and the ERP method, but the review led to a resetting of the risk premium and an adjustment to the formula used to update the ROE.

The OEB's current approach to cost-of-capital estimation requires that the Board determine a baseline ROE and subsequently update the estimate annually using the determined formula. The baseline ROE was most recently determined in 2009 during the generic proceeding. To arrive at its initial estimate of the ERP for determining the baseline ROE, the OEB reviewed the recommendations of the submissions as part of the 2009 proceeding, and determined each submission's Low, Medium, and High ERP.[91] In determining the initial ERP, the OEB found that:

> **the use of multiple tests to directly and indirectly estimate the ERP is a superior approach to informing its judgment than reliance on a single methodology.[92]**

As a result, the OEB considered all submissions, which included estimates based on the CAPM, DDM, risk premium model, econometric ERP analyses, realized ERP analyses, the difference between awarded ROEs and realized government bond yields, and various forecasts. The OEB averaged the experts' calculations of the risk premium over the long-term government bond and used judgment to determine that an appropriate premium over long-term government bonds was in the low-end of the range determined by the averages of the experts' ranges.

c) The U.K.

The U.K. regulator Ofgem has for many years made its cost of equity decisions within a CAPM framework, and, at least in a formal sense, has published CAPM parameters which correspond to its cost of equity determinations. However, it is also clear that Ofgem does not treat the CAPM estimates mechanistically, and, in any case, Ofgem uses a degree of judgment in determining the equity beta parameter, since there is little direct market evidence that can be relied on. While some of Ofgem's analysis and discussion of utility submissions is framed in terms of the CAPM parameters, it is clear that Ofgem focuses much more on the final cost of equity figure than on

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[91]   OEB 2009, p. 38.

[92]   OEB 2009, p. 36 (emphasis in the original).

the mechanistic derivation of that figure, whether in a CAPM framework or otherwise. For example, Ofgem has said: "Overall, our Final Proposals retain the cost of equity assumptions in our Initial Proposals of 7.0 percent for NGET and 6.8 percent for NGGT. Table 3.5 shows our Final Proposals for the cost of equity in terms of the CAPM components. We note, however, that **it is the overall allowed return that matters**. [emphasis added]"[93]

3. Impact of Economic, Industry or Company Factors

It makes sense that multiple cost of equity estimation methods have been developed and remain in use for a variety of reasons as articulated by Professors Berk and DeMarzo: "[a]ll the techniques … are imprecise" and "practitioners apply the technique that best suit their particular circumstances."[94] Because economic, industry, and firm-specific factors vary, it is important to assess the circumstances under which the models discussed in *Section III* are and should be used.

a) *Economic Factors*

As a pertinent example, due to the flight to quality following the financial crisis and subsequent monetary policy initiatives in many countries, the risk-free rate has been suppressed and is unusually low. Thus, in a standard implementation of the CAPM, the current risk-free rate results in a low cost of equity estimate. At the same time, investors have in recent years faced unusually high market volatility as measured by, for example, the S&P / ASX volatility index or the S&P 500 volatility index.[95] Academic literature finds that investors expect a higher risk premium during more volatile periods. For example, French, Schwert, and Stambaugh (1987) find a positive relationship between the expected market risk premium and volatility:

> We find evidence that the expected market risk premium (the expected return on a stock portfolio minus the Treasury bill yield) is positively related to the predictable volatility of stock returns. There is also evidence that unexpected stock returns are negatively related to the unexpected change in the volatility of

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[93] *RIIO-T1: Final Proposals for National Grid Electricity Transmission and National Grid Gas*, Ofgem, 17 December 2012, paragraph 3.45.

[94] Berk & DeMarzo 2009, p. 420.

[95] The S&P/ASX Volatility Index and the S&P 500 Volatility Index reflect the markets' expected volatility in the benchmark Australian and American equity indices, respectively.

stock returns. This negative relation provides indirect evidence of a positive relation between expected risk premiums and volatility.[96]

And Kim, Morley and Nelson (2004) find:

> When the effects of volatility feedback are fully taken into account, the empirical evidence supports a significant positive relationship between stock market volatility and the equity premium.[97]

Other academic papers have found a relationship between general economic conditions and the MRP. Constantinides (2008) studies a classical utility model where consumers are risk-averse and also summarizes some of the empirical literature. Empirical evidence shows that consumers become more risk-averse in times of economic recession or downturn, and equity investments accentuate this risk.[98] Increased risk aversion leads to a higher expected return for investors before they will invest. Specifically, equities are pro-cyclical and their performance is positively correlated with the economy's performance. Thus, unlike government bonds, equities fail to hedge against income shocks that are more likely to occur during recessions.[99] As a result, investors require an added risk premium to hold equities during economic downturns.

The very low current risk-free rates make the cost of equity estimates from a standard implementation of the Sharpe-Lintner CAPM also very low at a time when volatility measures indicate that the MRP has increased as well. Therefore, these market circumstances call for a serious consideration of economic factors or other models rather than a mechanical implementation of the Sharpe-Lintner CAPM.

Conditional models such as the Consumption CAPM attempt to incorporate the relationship between market volatility and the MRP in determining the cost of equity. As the model

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[96] K. French, W. Schwert and R. Stambaugh (1987), "Expected Stock Returns and Volatility," *Journal of Financial Economics*, Vol. 19, pp. 3.

[97] C-J. Kim, J.C. Morley and C.R. Nelson (2004), "Is There a Positive Relationship Between Stock Market Volatility and the Equity Premium?," *Journal of Money, Credit and Banking*, Vol. 36, p. 357.

[98] Constantinides, G.M. (2008), "Understanding the equity risk premium puzzle," In R. Mehra, ed., *Handbook of the Equity Risk Premium*, Elsevier, Amsterdam.

[99] Constantinides, G.M., and D. Duffie (1996), "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, pp. 219-240. See also E.S. Mayfield (2004), "Estimating the market risk premium," *Journal of Financial Economics*, vol. 73, pp. 465-496.

estimates a relationship between the risk premium of a stock and its conditional volatility, the model allows for a time-varying relationship between risk and return; *i.e.*, the implied cost of equity varies with the degree to which (i) the underlying stock can serve as a hedge against the market and (ii) market volatility. As rate-regulated entities commonly move with the market, the cost of equity estimate usually moves in the same direction as the volatility of the market. Thus, the consumption-based model addresses the finding that volatility impacts the required risk premium. As such, it may be particularly useful to implement this model when market volatility is unusually high or low.[100]

Given the currently very low risk-free rates and the recent market volatility, the DDM may additionally provide useful insights into the cost of equity. This is especially true for versions of the model that take into account (i) all cash that flows to shareholders through not only dividends but also share buybacks and (ii) changes in the forecasted growth rates in the near term and the longer term (*i.e.* multi-stage versions of the DDM).

Table 3 below displays the impact of two key economic factors discussed above, market volatility and risk-free rates, on the choice of cost of equity estimation model. While there is no specific formula that can be proposed to select a particular model under given market circumstances, or a method that can be used to combine the various models mechanistically, there are certain market scenarios under which it is more appropriate to use one model rather than another. For example, in times of either extremely high or low market volatility, (or extreme values of other macroeconomic indicators such as inflation), the consumption-based CAPM becomes more relevant. The DDM model and especially the multi-stage DDM is also less sensitive to variations in the risk-free rate than the standard CAPM, but it can be sensitive to market volatility. This is because in times of economic turmoil, the growth estimates for companies, including rate-regulated entities, are less likely to be stable going forward. Because the multi-stage DDM has more realistic characteristics and is less sensitive to analysts' short-term forecasts, the tables in this section use the term DDM to reflect the multi-stage DDM.

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[100] See Ahern, *et al.* (2012) for a discussion of its use in a regulatory setting.

The effect of the risk-free rate and market volatility on model choice is reflected in Table 3 below, which should be viewed as an illustration on the directional choice rather than a prescription.

Table 3: Relationship Between Key Economic Conditions and Weights to be Given to Models

		Prevailing Risk-free Rate in Economy		
		High	**Average**	**Low**
Market Volatility	**High**	Consumption CAPM		
	Average	Consumption CAPM / DDM	CAPM / ECAPM	Consumption CAPM / DDM
	Low	Consumption CAPM / DDM		

b) *Industry Factors*

As discussed above, empirical research has consistently found that the Security Market Line determined by the Sharpe-Lintner CAPM (as depicted in Figure 2) is too steep.[101] This result is also consistent with the findings of Fama & French (1992), which estimated a zero slope in the empirical SML.[102] Thus, the ECAPM as well as the Fama-French model attempt to find a model that is a better fit with empirical data from tests of the Shape-Lintner CAPM, showing that the latter tends to under estimate the cost of equity for companies with beta estimates below one, and over estimate the cost of equity for companies with beta estimates above one. A better-fitting model flattens the Security Market Line as depicted in Figure 3. Because most rate-regulated entities have beta estimates below one, reliance on the Sharpe-Lintner CAPM tends to bias the

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[101] See, for example, F. Black, M.C. Jensen, and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," *Studies in the Theory of Capital Markets*, Praeger Publishers, 1972, pp. 79-121 and E.F. Fama and J.D. MacBeth, "Risk, Returns and Equilibrium: Empirical Tests," *Journal of Political Economy* 81 (3), 1972, pp. 607-636.

[102] E.F. Fama and K.R. French, "The Cross-Section of Expected Returns," *Journal of Finance* 47, 1992, pp. 427-465.

cost of equity estimates for these companies downwards. Therefore, for entities whose beta estimates are farther from one, it becomes important to look to the ECAPM to accurately reflect the cost of equity for the entity.[103] In many countries or regions, including Australia, Canada, Europe and the U.S, estimated betas for rate-regulated entities declined and become statistically insignificant in the early 2000s as the dot.com bubble burst. In such circumstances, the downward bias in the cost of equity estimates from the Sharpe-Lintner CAPM becomes more pronounced and models such as the ECAPM can improve the estimation.

For some industries the future may look like the past, but for others this is not the case. As an example, the outlook for the U.S shale gas industry today is different than it was in 2008. Similarly, the outlook for the nuclear industry in Japan changed dramatically after the 2011 tsunami. In such circumstances, forward-looking estimates of the industry's cost of capital as obtained through, for example, versions of the DDM, may be especially useful. As noted above, the DDM implementation should carefully consider not only the current economic environment but also industry and firm-specific factors, such as the sustainability of the current growth forecasts and whether dividends truly reflect all cash distribution to shareholders. For example, the multi-stage models discussed in *Section III* rely on several growth rates and therefore enable the analyst to consider near-term, intermediate, and long-term growth prospects for the individual company, industry, and economy. Therefore, a multi-stage DDM model, unlike the Sharpe-Lintner CAPM, can capture both near-term and longer-term changes in an industry. This becomes especially important when an industry's expected risk characteristics differ from its past characteristics.

Rate-Regulated Entities vs. Other Industries

According to empirical studies, the Sharpe-Lintner CAPM remains the most commonly used model across the full spectrum of companies.[104] However, the utility industry and rate-regulated entities have some unique characteristics that make it plausible that the methods that serve other

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[103] See Table A-1 in the Appendix for details. Much of the academic literature estimating alpha dates back to the 1980s. Academic research has since turned to the Fama-French multifactor model, which attempts to explicitly capture the empirical pivot of the SML as a function of additional pricing factors.

[104] J.R. Graham and C.R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60, 2001, pp. 187-243.

industries well do not serve this industry nearly as well. For example, the utility industry tends to be relatively stable, so that the DDM (and especially the multi-stage DDM) is much more likely to provide usable results for this industry than for more volatile industries.   As the residual income valuation model is a variation of the multi-stage DDM, the same comments pertain to this model.

Prior to the financial crisis, models such as the single-stage DDM, *Brattle*'s multi-stage DDM, the CAPM, and versions of the ECAPM resulted in fairly similar results. Figure 5 below illustrates this for the gas distribution industry in the U.S. towards the end of 2006. Specifically, the figure is based on implementing the constant growth DDM, a 3-stage DDM, the Sharpe-Lintner CAPM, the ECAPM with an alpha of 0.5% and an ECAPM with an alpha of 1.5% for seven gas distribution companies. Figure 5 then shows the range of the cost of equity estimates assuming a 50-50 gearing for the target company. The figure also indicates the average cost of equity obtained from the sample, which is at the split of each bar.
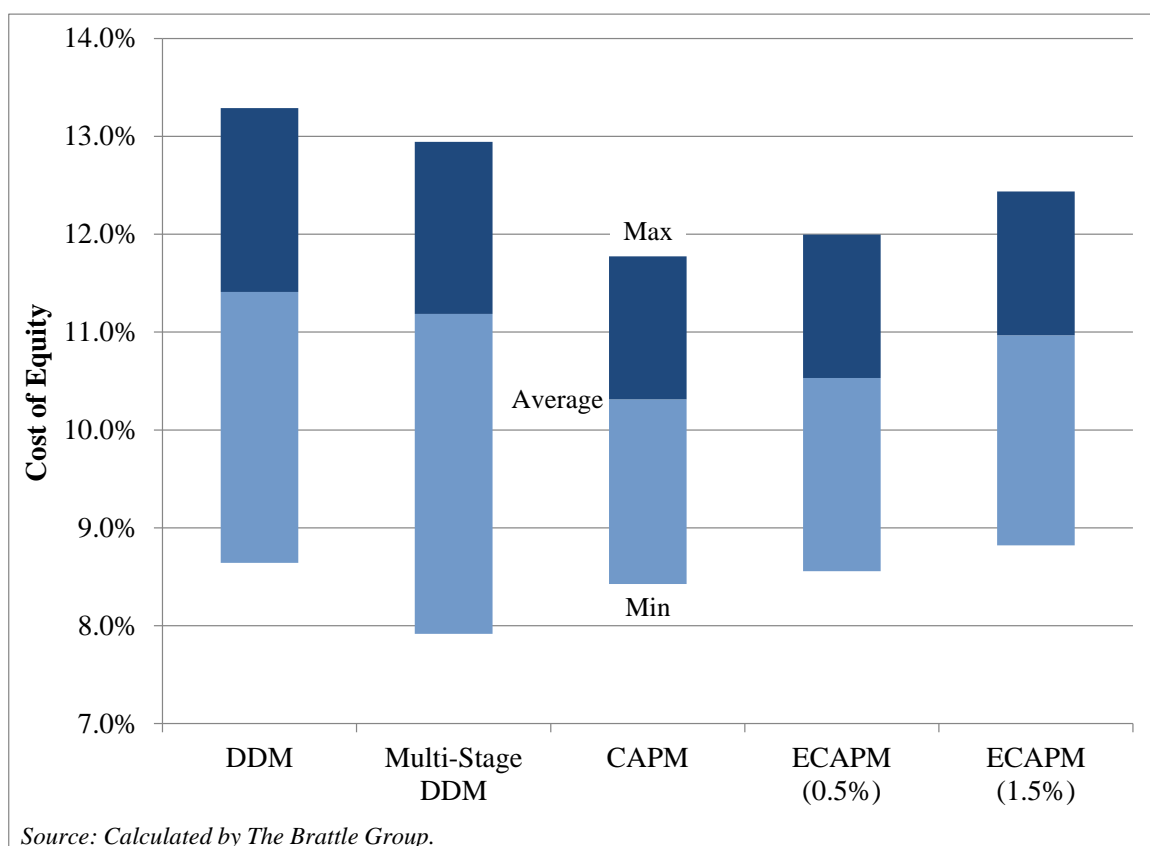
Figure 5

It is clear from Figure 5 above that there is substantial overlap in the estimates. We attribute this effect to the fact that the economy was relatively stable in 2006 and so was the gas distribution industry. At the time, these models largely confirmed the range of the cost of equity estimates.

As discussed above, rate-regulated companies also tend to be low-beta entities, so the empirical finding that the SML predicted by the Sharpe-Lintner CAPM is too steep is a serious concern for this industry; *i.e.*, it becomes important to use the ECAPM or other models to ensure that this empirical observation is accurately reflected in the cost of equity estimates.

Analogously to Table 3, Table 4, Panels A and B below summarize the directional weighing of the models depending on various industry characteristics. The two industry factors considered in Table 4, Panel A below are the stability of growth rate forecasts and the average market beta of the industry. For example, as mentioned above, rate-regulated entities tend to have relatively more stable growth forecasts over time and low betas (*i.e.*, beta estimates below one). Therefore, for this industry, the use of the ECAPM or variations of the multi-stage DDM might become valuable in determining the cost of equity capital. The effect of the stability of growth forecasts and the beta value on model choice is reflected in Table 4, Panel A below, which should be viewed as an illustration on the directional choice rather than a prescription.

Table 4: Relationship Between Key Industry Factors and Weights to be given to Models–Panel A

		Stability of Industry Growth Forecasts Over Time		
		High	**Average**	**Low**
Market Beta of Industry	**High**	ECAPM / DDM		ECAPM / Other
	Average	CAPM / DDM		CAPM / Other
	Low	ECAPM / DDM		ECAPM / Other

Another characteristic of the industry that should be considered is whether companies in the industry are exposed to financial distress and/or significant merger and acquisition activity, and

the prevalence of share buybacks. As discussed above, market-based estimation models are relatively more affected when a given company faces financial distress, or unique circumstances that may lead to its stock price decoupling from fundamentals. Therefore, if many companies in an industry are subject to such effects, the whole industry may be affected. Further, companies that engage in a substantial amount of share buybacks will end up distributing cash to shareholders in a form other than dividends, which makes a DDM based on a per share dividend ratio less appropriate. Panel B below illustrates these effects.

Table 4: Relationship Between Key Industry Factors and Weights to be given to Models– Panel B

		Industry Exposure to Financial Distress and/or M&A	
Prevalence of Share Buybacks		**High**	**Low**
	High	Other Models: Risk Premium, comparable earnings, maybe use other industries	CAPM, ECAPM, DDM that includes all cash that accrues to shareholders
	Low		CAPM, ECAPM, DDM

c)   *Company Factors*

In many instances company-specific issues are better dealt with via sample selection or through risk positioning than through the determination of how to estimate the cost of equity. A company that is a potential member of the benchmark sample is often dropped if it faces unique circumstances that may bias the cost of capital estimation process. This is the case if, for example, a company is undergoing significant merger or acquisition activity, which inherently affects the information available in the market and therefore drives the stock price (and thus the results from all market-based models, including the CAPM, ECAPM, Fama-French and DDM).

After a range of cost of equity estimates has been obtained, it is necessary to consider where, from within this range, the final determination on the cost of equity will be. Provided that the range has been developed in an appropriate way that takes account of the market and industry factors described in this section, the final step is to consider the relative risk of the target company compared to the sample of companies from which the cost of equity range has been developed. The cost of equity is adjusted upward or downward depending on the target entity's risk characteristics relative to those of the sample. This issue is the topic of the next section.

D.   RISK POSITIONING OF THE TARGET ENTITY

The discussion in the preceding sections covered various models that produce cost of equity estimates. Typically the cost of equity will be estimated for a sample of firms, or all firms in a particular sector, because it usually is not possible to estimate the cost of equity for a single firm with a useful degree of accuracy. To determine a single value for the cost of equity for a specific firm from a range of values for a set of comparator firms, it makes sense to consider the riskiness of the specific firm relative to the riskiness of the sample, since the cost of equity itself is compensating investors for risk.

In the regulatory context, in some cases this process is implicit in the regulator's decision, while in others it is an explicit step in the cost of equity determination process. This step can conveniently be termed "risk positioning", because the regulator considers the risk characteristics of the specific utility relative to the benchmark.

1.   Why risk positioning is necessary

While the precise details and wording of the regulator's objective in setting the cost of equity vary from one jurisdiction to another, the underlying idea is that investors will expect a return equivalent to the return that they would expect from other investments of like risk. Utilities generally have low risk relative to the market as a whole, but within the utilities sector, different firms are likely to have somewhat different risk characteristics. "Risk positioning" acknowledges the possibility that different utilities can have somewhat different risks. In this context, "risk" is defined as the characteristic of an investment which determines expected returns which would usually include "systematic" exposure to the wider economy, but not "idiosyncratic" risks associated with specific projects that can be diversified away in an investment portfolio. While

the cost of equity solely captures investors' compensation for bearing systematic risk, the cost of debt reflects total risk, including idiosyncratic risks. Therefore, there are instances of regulatory mechanisms, such as decoupling, which reduce the variability of total revenues and therefore also total risk, (affecting the cost of debt), but which may not impact the cost of equity for a given utility.

One way in which a utility is exposed to systematic risk is through variations in demand. End-user demand tends to be at least somewhat correlated with wider economic activity, and is thus a source of exposure to systematic risk. One utility might have more exposure than other, for example if it has a greater proportion of price-sensitive industrial load.

In some jurisdictions, leverage is considered a source of "financial risk", which affects the risk positioning analysis. This could be so, for example, where the rate of return is generally determined on the basis of actual capital structure. A utility with more debt than the benchmark will require a higher return on equity than the benchmark, even if it otherwise has similar business risk exposure as the benchmark (just as two utilities with the same asset beta would have different equity betas if one has higher gearing than the other). Where this approach is taken, the term "business risk" is used to refer to the other sources of relevant risk differences that are taken into account in the risk positioning analysis.

Once a benchmark rate of return has been defined (whether a point estimate or a range), the risk positioning approach requires an analysis of the particular utility's risk relative to that benchmark. To the extent that the utility is found to have more (or less) risk than the benchmark, the rate of return would be set higher (or lower) than the benchmark rate of return.

2.   What risk characteristics are relevant?

The characteristics relevant to risk positioning are those which expose the utility to systematic risk and which therefore have an impact on the rate of return required by investors. Some important sources of uncertainty in revenues and returns to investors may not have an impact on the required return to the extent that investors are able to diversify away exposure to those risks. For example, the weather may be an important source of variability in revenues and returns, but may not be an important source of risk to investors because it is diversifiable.

A good way to think about risk positioning is to consider the extent to which different utilities are protected from risks. A distribution utility can in principle be protected from risks to the extent that it is able to pass on risk to its customers (which depends on the detail of the regulatory framework being applied). Demand risk (which is at least partly non-diversifiable), for example, can be borne by the utility if the regulatory regime sets prices and does not "true up" revenues to account for the difference between forecast and actual demand. Alternatively, demand risk can be passed on to customers through a true-up or balancing account process, which would allow the utility to recover in one year any "missing" revenue from the prior year caused by demand forecasting errors. Protection from demand risk in this way depends on both a regulatory framework that allows for such true-ups and on the existence of franchise customers that will bear the risks passed on to them. Therefore, other things equal, a utility with true-ups for demand risk would be considered less risky than one without.

Distribution utilities typically have franchise customers that rely on the utility and have no alternative supply of energy. However, this is typically not the case for gas pipelines: in many jurisdictions, gas pipelines do not have "franchise" customers: customers may be free to switch to competing pipelines. Even if there is no prospect of competition from other pipelines, it may still be difficult for pipelines to pass on demand risk to their customers, since large end-users may be price sensitive (i.e., if the pipeline increases price in response to a fall in demand, the price increase itself could further cut demand).

Pipeline regulators in both the US and Canada apply a risk-positioning approach in determining the cost of equity.

3. FERC Approach

The Federal Energy Regulatory Commission (FERC) has a standard approach to determining the cost of equity for gas pipelines, set out in a "policy statement",[105] which, together with precedent from prior decisions, guides all decisions on the cost of equity for gas pipelines. The FERC's approach is to use a form of the dividend growth model (typically termed the "DCF" model in

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[105] *Composition of Proxy Groups for Determining Gas and Oil Pipeline Return on Equity*, FERC (April 2008).

the US) to estimate the cost of equity for a benchmark group of publicly-traded pipeline companies. The results of the model are a cost of equity estimate for each of the companies in the benchmark (or "proxy") group.

FERC starts by assuming that the median company in the proxy group is the appropriate cost of equity, unless either the pipeline or an intervener in the case demonstrates that the instant pipeline has risk factors which mean that the cost of equity should be set above or below the median:

> *after defining the zone of reasonableness through development of the appropriate proxy group for the pipeline, the Commission assigns the pipeline a rate within that range or zone, to reflect specific risks of that pipeline as compared to the proxy group companies.* [f/n omitted] *The Commission has historically presumed that existing pipelines fall within a broad range of average risk. A pipeline or other litigating party has to show highly unusual circumstances that indicate anomalously high or low risk as compared to other pipelines to overcome the presumption.*[106]

And

> unless *a party makes a very persuasive case in support of the need for an adjustment and the level of the adjustment proposed, the Commission will set the pipeline's [ROE] at the median of the range of reasonable returns.*[107]

In line with this approach, most FERC decisions result in the pipeline receiving a cost of equity equal to the median of the proxy group. A recent decision for El Paso Natural Gas (EPNG),[108] however, illustrates how FERC assesses relative risk and may, on occasion, move away from the median. In this case, the FERC ALJ[109] characterized EPNG's business risk on two related dimensions: competitive risk and regulatory risk. US natural gas pipelines typically secure long-term contractual commitments from shippers to use the pipeline capacity (with relatively high fixed charges, equivalent to a take-or-pay commitment). EPNG had long-term contracts for a smaller proportion of its capacity than did the pipelines in the proxy group, and its contracts were typically shorter. Furthermore, EPNG's throughput had been declining. This is symptomatic of higher business risk, because in the absence of contractual commitments and in the absence of

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[106] *Ibid.*, p. 4.

[107] *El Paso Natural Gas Company*, Initial Decision, docket no. RP10-1398 (June 18, 2012), paragraph 40, quoting prior FERC decisions.

[108] *El Paso Natural Gas Company, Initial Decision*, docket no. RP10-1398 (June 18, 2012).

[109] A FERC rate case typically results in an "initial decision" issued by an Administrative Law Judge (ALJ). The ALJ's decision is subsequently reviewed by the FERC commissioners, and may be affirmed or varied.

franchise customers, the pipeline is no longer able to pass on risks to its customers. In the limit, the pipeline may be unable to charge rates high enough to recover its authorized revenue requirement (as increasing rates drives throughput lower still).

The ALJ found that EPNG was exposed to competition in its major downstream markets from new pipeline projects, and that this competition was to an extent the result of regulatory policies that favor new pipeline projects to foster competition (possibly harming existing pipelines).

Based on this analysis (and also a finding that EPNG had above-average financial risk, as evidenced by a credit rating of BBB-, lower than all but one of the proxy group companies), the ALJ determined that EPNG's cost of equity should be set "well above the median ROE [of the proxy group]".[110]

4.   NEB approach

In Canada, the approach taken by energy regulators (both provincial and national) historically was to set the cost of equity on a formula basis and to us the same cost of equity for all pipelines. Risk positioning was then used to vary the authorized proportion of equity in the capital structure, thereby increasing the overall return on capital for those utilities judged to be riskier. However, in the most recent decision by Canada's National Energy Board (NEB), the NEB moved to an approach which focuses on the overall after-tax return directly, rather than separately determining the cost of equity, the cost of debt, and the proportion of each in the capital structure.[111] The NEB takes a systematic approach to assessing business risk under the headings "supply risk", "market [downstream] risk", "regulatory risk", "competitive risk" and "operating risk", although the NEB said "The various forms of risk are in some cases inextricably linked, and the boundaries between them are subjective".[112] In the RH-1-2008 case,[113] the NEB was concerned with whether the business risk of the pipeline had increased

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[110] *Ibid.*, p. 45. The ALJ did not specify an ROE. The final decision on ROE rests with the FERC commissioners.

[111] See RH-1-2008, discussed further below.

[112] *Reasons for Decision, Trans Quebec and Maritimes Pipelines Inc., RH-1-2008*, NEB (March 2009), p. 30.

[113] Concerning the Trans Quebec and Maritimes Pipelines, which predominantly move supplies sourced from the Western Canadian Sedimentary Basin (WCSB) via the TransCanada Mainline, into Quebec and on into New Hampshire.

since the last time that a decision on the cost of capital for the pipeline had been taken. The NEB identified a number of factors as contributing to an increased overall business risk.

- **Supply risk**: the pipeline was mainly supplied from a region with declining conventional production and rising costs. While it was possible that new sources of unconventional supply (shale gas) would be developed, the result was increased uncertainty over the availability of competitively-priced supplies, and hence concerns over the possibility for reduced throughput.

- **Market and competitive risk**: because a large and increased proportion of the pipeline's throughput went to large industrial and electric power generation load, which is more variable than domestic and commercial load. In addition, competition with cheap hydro-power in the Quebec also contributed to increased market risk. Market risk was also increased as a result of the potential for competition with LNG imports in the US market.

Overall, the NEB concluded that business risk had increased as a result of these factors relative to the previous cost of capital decision for the pipeline. Whereas the FERC in the US uses a risk positioning approach to determine the cost of equity relative to a benchmark, the NEB estimated the after-tax weighted average cost of capital directly, principally on the basis of market-based estimates of the cost of capital of various comparator companies. The business risk analysis described above was part of the NEB's determination of where the pipeline's cost of capital should be relative to the sample data.[114]

5. Implementation

In the FERC and NEB examples given above, risk positioning of the target utility within the range of comparator or proxy companies is not analytically precise: the regulator considers evidence (which could be quantitative, such as the proportion of price-sensitive industrial load, or more qualitative) as to exposure to various relevant risk factors. Weighing the risk factors, and determining how the analysis of risk should be reflected in the final cost of equity determination is necessarily imprecise, and relies on judgment. For example, a regulator might determine that a

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[114] The NEB's analysis is summarized on p.79 of the decision.

particular utility, having an unusually high proportion of industrial load, was of above average risk, and that as a result the cost of equity should be 50 basis points above the mid-point of a range determined for a sample of utilities. The direction of the adjustment (upwards) is clear, but the magnitude is more a matter of judgment than something that can be derived quantitatively.

APPENDIX: ADDITIONAL TABLES AND FIGURES

Table A-1: Empirical Evidence On The Alpha Factor in ECAPM

AUTHOR	RANGE OF ALPHA	PERIOD RELIED UPON
Black (1993)[1]	1% for betas 0 to 0.80	1931-1991
Black, Jensen and Scholes (1972)[2]	4.31%	1931-1965
Fama and MacBeth (1972)	5.76%	1935-1968
Fama and French (1992)[3]	7.32%	1941-1990
Fama and French (2004)[4]	N/A	
Litzenberger and Ramaswamy (1979)[5]	5.32%	1936-1977
Litzenberger, Ramaswamy and Sosin (1980)	1.63% to 3.91%	1926-1978
Pettengill, Sundaram and Mathur (1995)[6]	4.6%	1936-1990

[*] The figures reported in this table are for the longest estimation period available and, when applicable, use the authors' recommended estimation technique. Many of the articles cited also estimate alpha for sub-periods and those alphas may vary.

[1] Black estimates alpha in a one-step procedure rather than in an un-biased two-step procedure.

[2] Estimate a negative alpha for the sub period 1931-39 which contain the depression years 1931-33 and 1937-39.

[3] Calculated using Ibbotson's data for the 30-day treasury yield.

[4] The article does not provide a specific estimate of alpha; however, it supports the general finding that the CAPM underestimates returns for low-beta stocks and overestimates returns for high-beta stocks.

[5] Relies on Litzenberger and Ramaswamy's before-tax estimation results. Comparable after-tax alpha estimate is 4.4%.

[6] Pettengill, Sundaram and Mathur rely on total returns for the period 1936 through 1990 and use 90-day treasuries. The 4.6% figure is calculated using auction averages 90-day treasuries back to 1941 as no other series were found this far back.

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