The Brattle Group

Some Theory on
Price and Volatility Modeling

by

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Introduction and Overview

To get the benefits of a risk analysis tool, managers and regulators must have a basic understanding of its analytic foundations. Users also must know how design choices affect results.

Key elements of electric or gas price risk management modeling:

- Typical price distribution: lognormal
- Term structure of volatility: the two-factor model
  - Mean reversion
  - Seasonality
- Simulating forward curve dynamics: Monte Carlo
- Calculating future option premiums
- Calculating and summarizing total cost distributions
- Assessing cash and credit risk to the utility

Price Mean and Uncertainty

Forward price of a commodity $F_{t,T}$ will change over procurement horizon in relation to expected spot price P_T

- Small "t" subscript refers to current transaction date (trading day)
- Big "T" subscript refers to future delivery or settlement date

F price changes randomly with new information up to settlement date

- F at delivery date is spot price, *i.e.* $F_{T,T} = P_T$
- F price treated in model (as is common practice) as an unbiased estimate of future spot, i.e. no trend
- Process of F evolving over time is modeled as variation on a "random walk" in a risk-neutral environment

Probability Distribution of Prices

Commodity prices are typically asymmetric, in the sense that they are non-negative and have more upside potential than down-side.

We model percentage changes in forward prices $F_{t,T}$ over short time intervals having a normal distribution

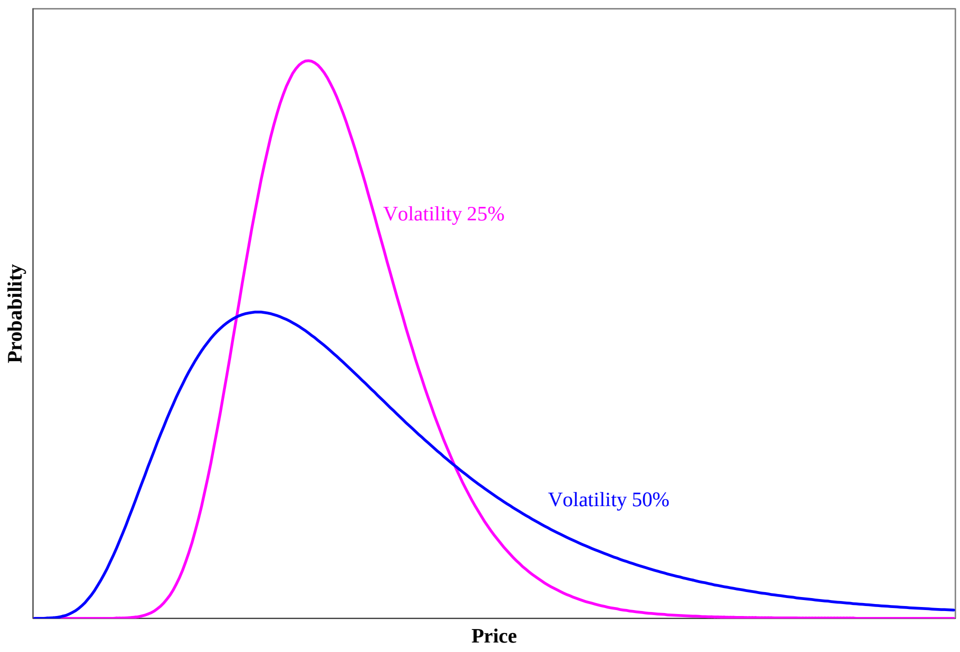
Then future spot prices P_T will have a lognormal distribution

- Empirically found to describe distribution of many commodities, including natural gas

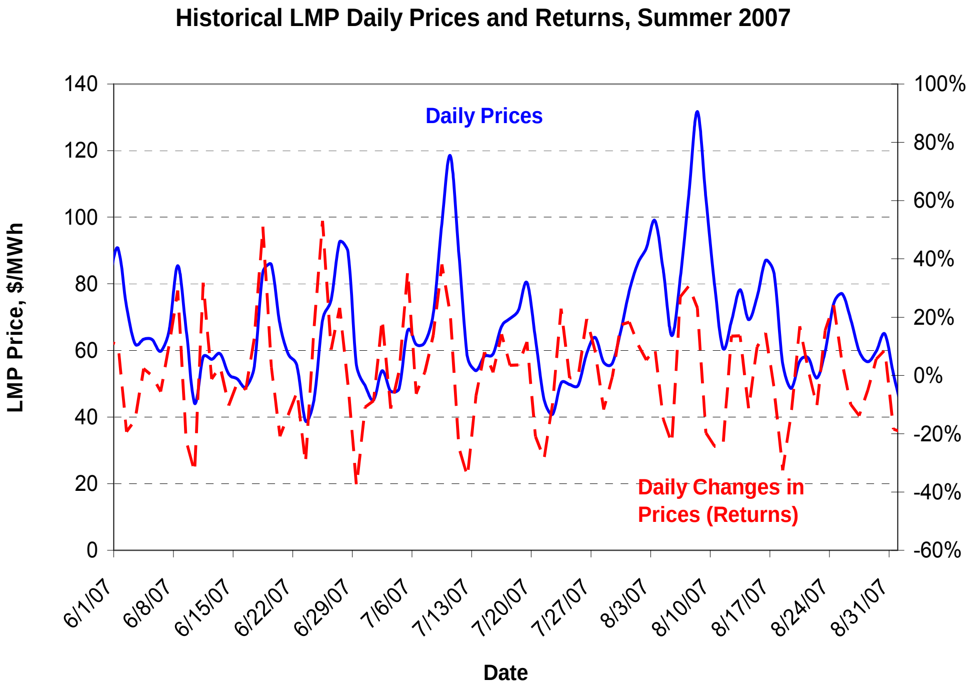
Properties of Lognormal Distribution

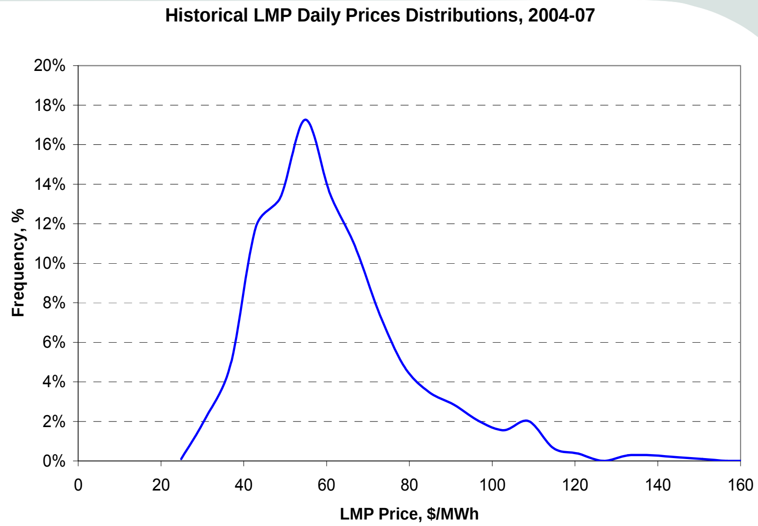
Lognormal Distribution:

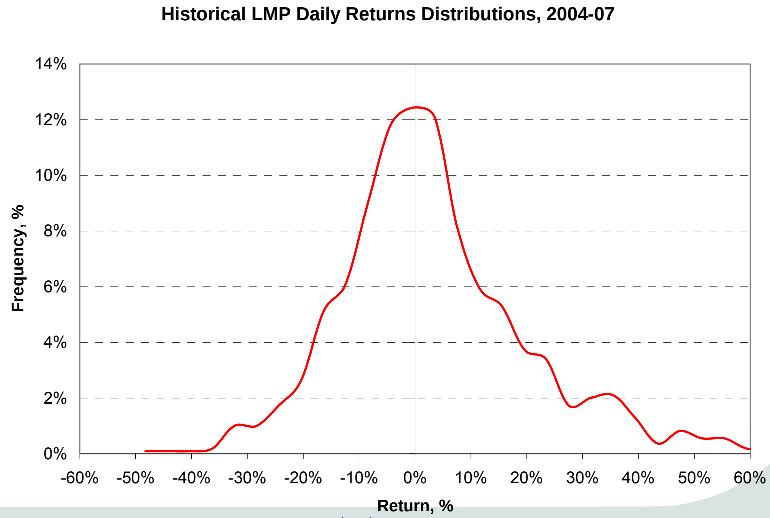
- Has the right shape
  - Bounded on the lower end by zero
  - Skewed to the right (no theoretical upper limit on prices)
- Up or down <u>percentage</u> price movements (returns) are equally likely
- Completely described by two parameters: mean and variance
  - Can be enhanced to capture other time patterns (see next)



PJM Price Histories and Distributions



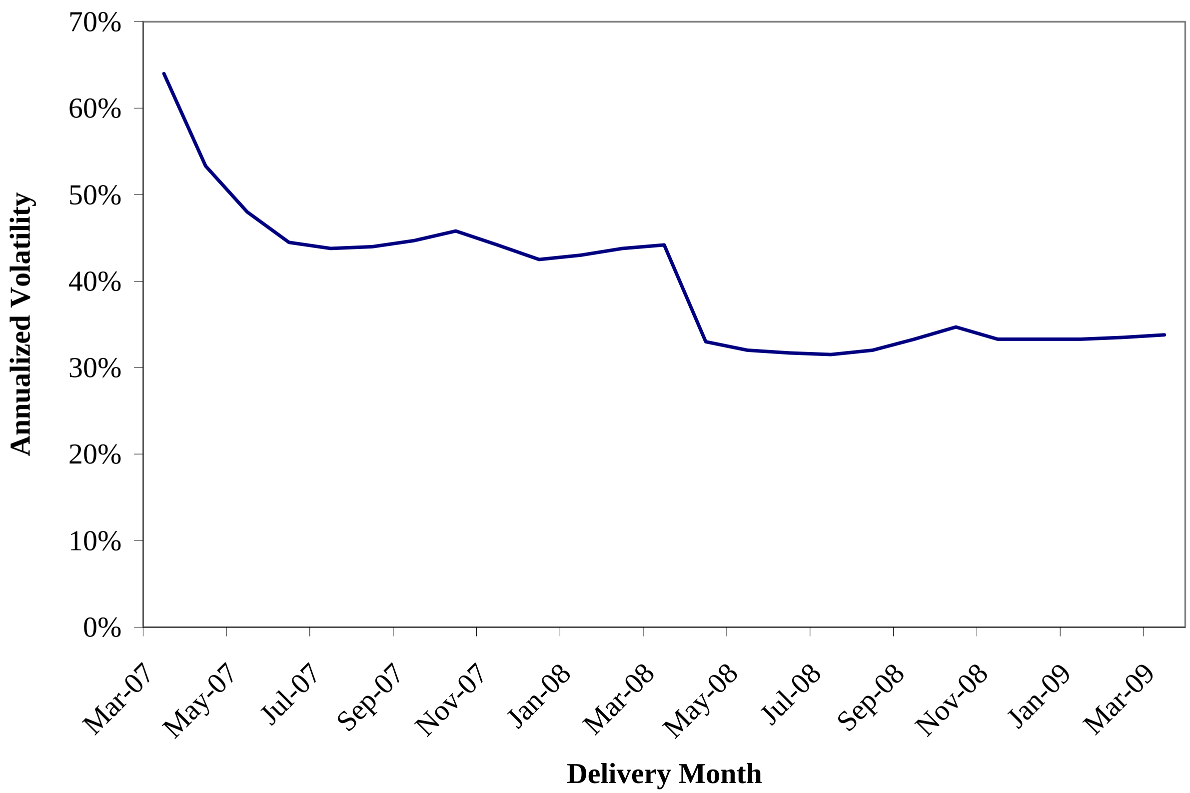




Volatility Term Structure

Volatility term structure refers to the relation between volatility and delivery time T.

- Typically, term structure of annualized volatilities is declining (but not smoothly)
  - High near-term volatility that decays to a steady, lower long-term level
  - May exhibit seasonality (some months usually more volatile than others)
- Seasonality can vary by trading date as well as by delivery date



Quoted volatilities implied by contracts for natural gas call options (available from brokers) as of February '07

Volatility Model: Two-Factor Model

Volatility term structure can be modeled as a mean-reverting, two-factor model.

- Forward curve response to new information is subject to "mean reversion" (declining influence into future) represented as

$$\sigma(t,T) = \beta_S \, e^{-\eta(T-t)}$$

where β_S is the "short volatility" t is current date

η is the "mean reversion rate" T is settlement date

- "Long volatility" for delivery in distant months tends to level off at a positive level, β_L ,reflecting uncertainty in long-run marginal costs
- Overall volatility is a combination of both; these information types are independent, so their variances add:

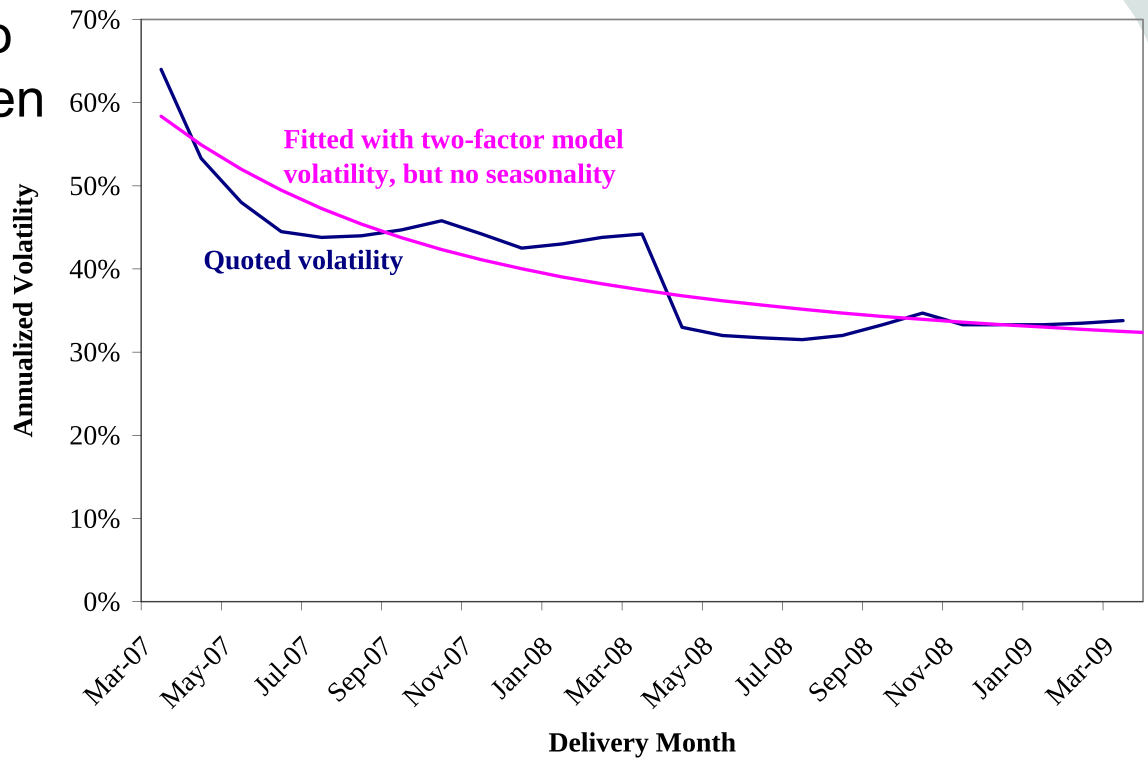
$$\sigma(t,T) = \sqrt{\beta_S^{\,2} \, e^{-2\eta(T-t)} + \beta_L^{\,2}}$$

Fitting the Two-Factor Model

The parameters of the two factor model can be chosen to "best-fit" quoted volatilities:

$$\sigma(T) = \sqrt{\frac{1}{T}\int_0^T \sigma^2(t,T)dt}$$

$$= \sqrt{\beta_S^2\left(\frac{1-e^{-2\eta(T-t)}}{2\eta(T-t)}\right)+\beta_L^2}$$



Fitted values:

<u>short volatility</u> = β_S = 57% / year

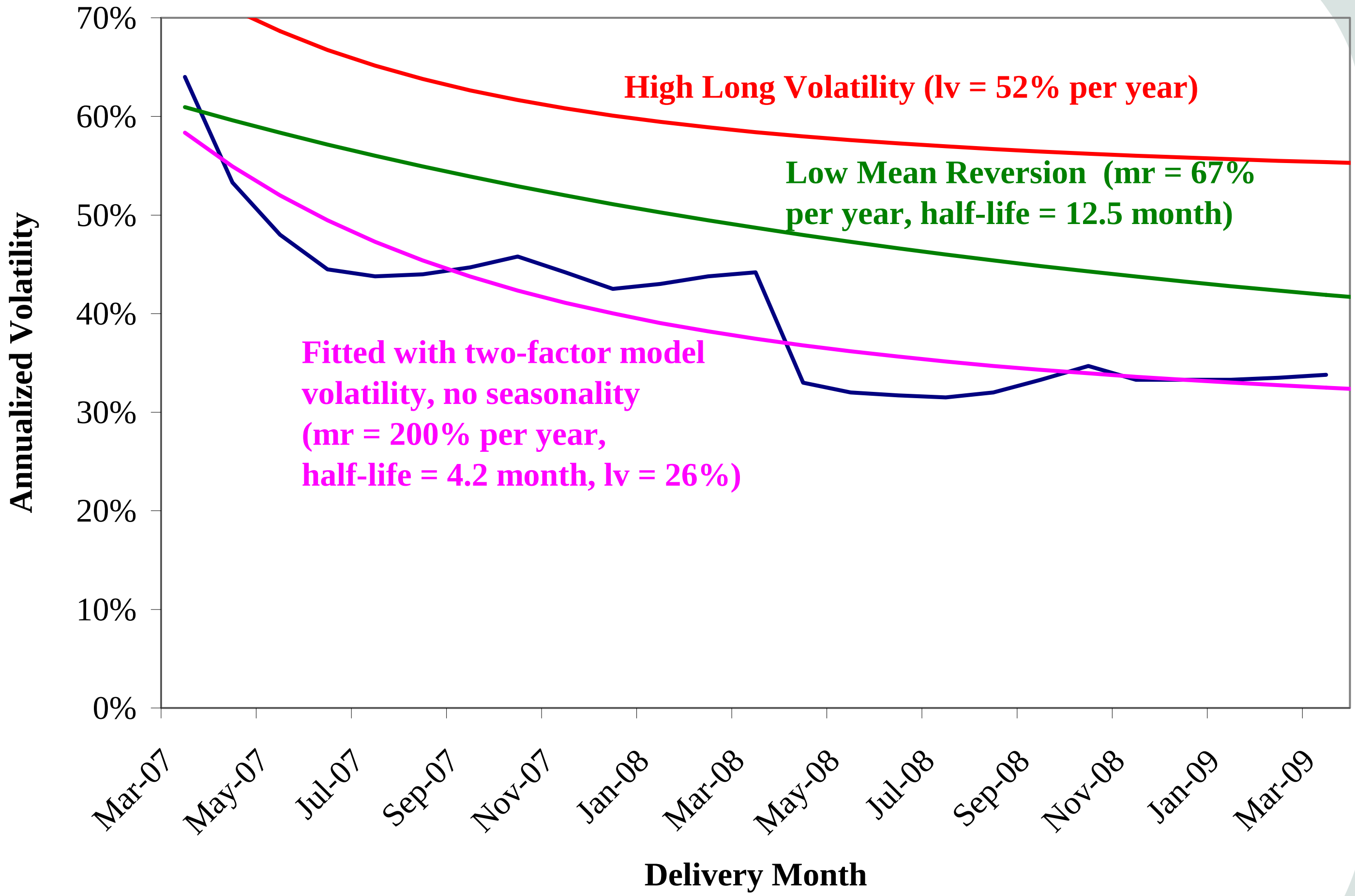
<u>mean reversion</u> = η = 200% / year has a half-life of four months, i.e. it takes four months for the volatility shock to die out to half of its original value

<u>long volatility</u> = β_L = 26% / year

Understanding the Impact of the Two-Factor Model Parameters

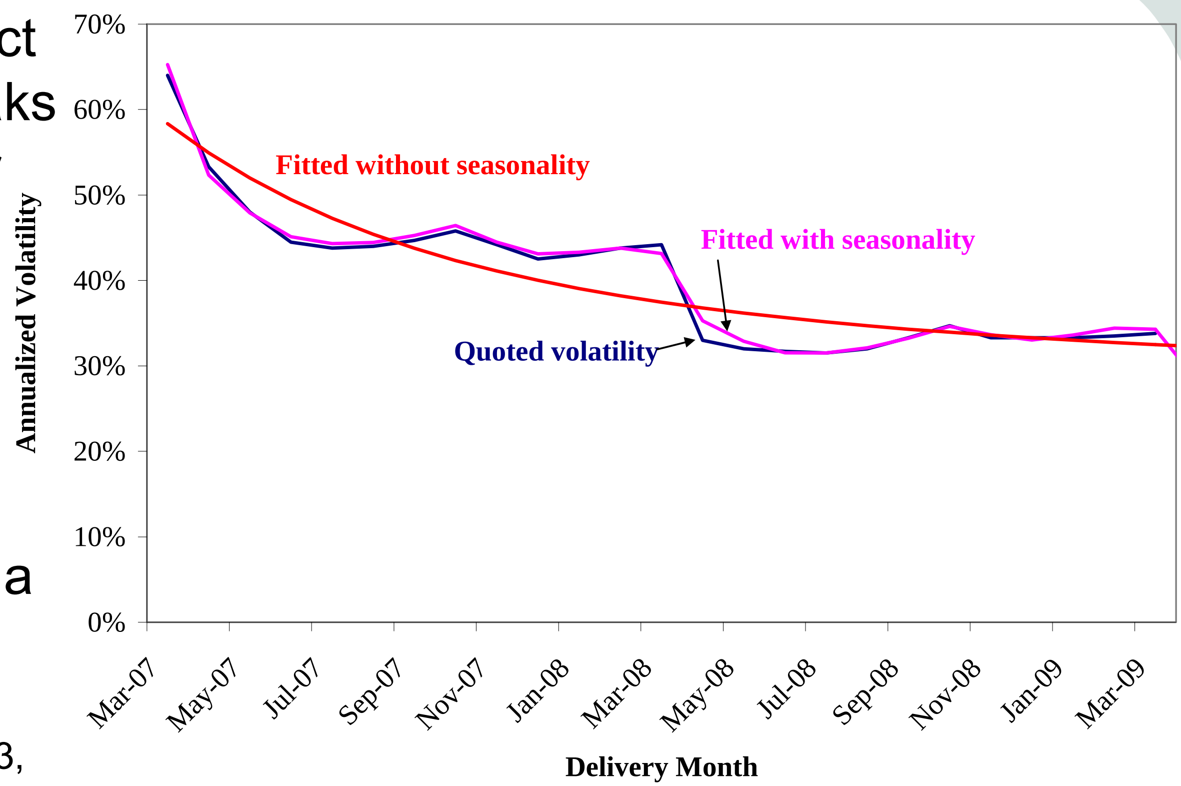
Higher long-factor volatility (e.g., twice the original value) causes higher long term volatility level

Lower short-factor mean reversion (e.g., one third of the original value) slows the decay of volatility to its long-term level



Adding Seasonality to the Two Factor Model

The volatility with respect to delivery time has peaks and valleys at particular delivery seasons.



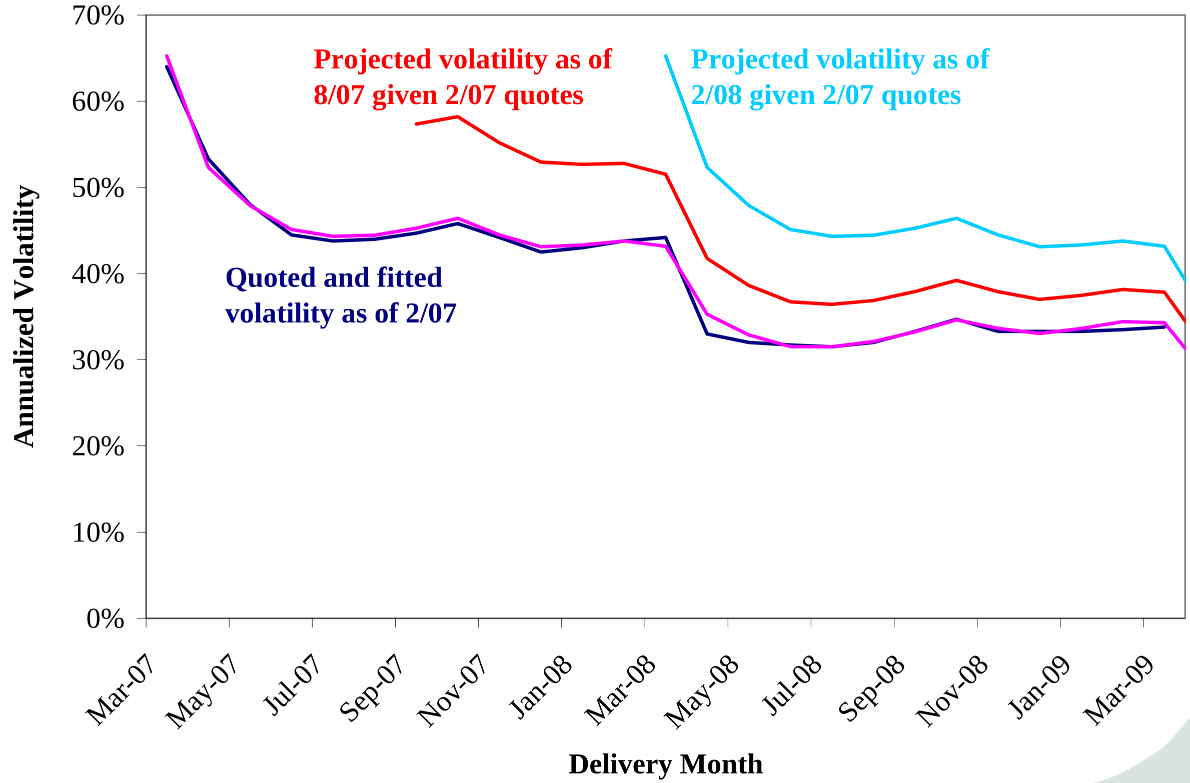
We capture delivery seasonality by scaling average volatility up or down in each month by a factor:

Jan = 1.07, Feb = 1.11, Mar = 1.13, Apr = 0.94, May = 0.90, Jun = 0.88, Jul = 0.90, Aug = 0.94, Sep = 0.99, Oct = 1.05, Nov = 1.04, Dec = 1.04

Simulating Future Volatility Using Two-Factor Model

From the parameter estimates of the two-factor model, we can simulate how future average volatility will shift, in a fashion consistent with the original quotes.

- New t causes near-month volatility to increase
- Also different T-t remaining, and must apply appropriate seasonality factors



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Simulating Forward Curve Dynamics

The next step is to simulate many possible future forward curves created by sampling from the volatility equation and perturbing the original forward curve accordingly.

In the two-factor model, forward price changes occur in response to two types of information: “short-term” and “long-term”

short and long volatility coefficients

$$\%\Delta F_{t,T} \approx \frac{dF_{t,T}}{F_{t,T}} = \beta_S e^{-\eta(T-t)} dz_t^S + \beta_L dz_t^L$$

mean-reversion coefficient   random variables in simulations

Simulating Forward Curve Dynamics

Goal: simulate forward prices $F_{T,T+i}$ i=1,2,… at trading date T for delivery at T+1,T+2 etc. (outlook as of the current time, t)

1. Simulate uncorrelated standard normal random variables dz_t^S and dz_t^L representing "draws" of new short-term and long-term information (factors) in period t.
2. Compute cumulative uncertainty for each factor from the current time, t, to delivery time, T.
3. Compute future spot price P_T consistent with cumulative price uncertainty and prevailing forward curve.
4. Compute future forward price $F_{T,T+1}$ perturbing the original forward curve $F_{t,T+1}$ using the forward price dynamics: exponentially decay short factor at its mean reversion rate and do not decay long factor.
5. Repeat thousands of times, such that average of simulations recapitulates the original forward curve and volatility quotes.

Simulated Future Spot Prices

Recall that if changes in forward prices are normally distributed, we can write forecasts of future spot prices in the standard lognormal form:

$$P_T = F_{0,T} \exp\left(w_{L,T} + w_{S,T} - \tfrac{1}{2}\overline{\sigma}(T)^2 T\right)$$

Mean correction of lognormal distribution

where $\tilde{Z}$ is a standard normal

$\overline{\sigma}(T)$ is cumulative volatility from trading date *t* to delivery date *T*

w_S, w_L are short and long volatility factors following dynamic equations:

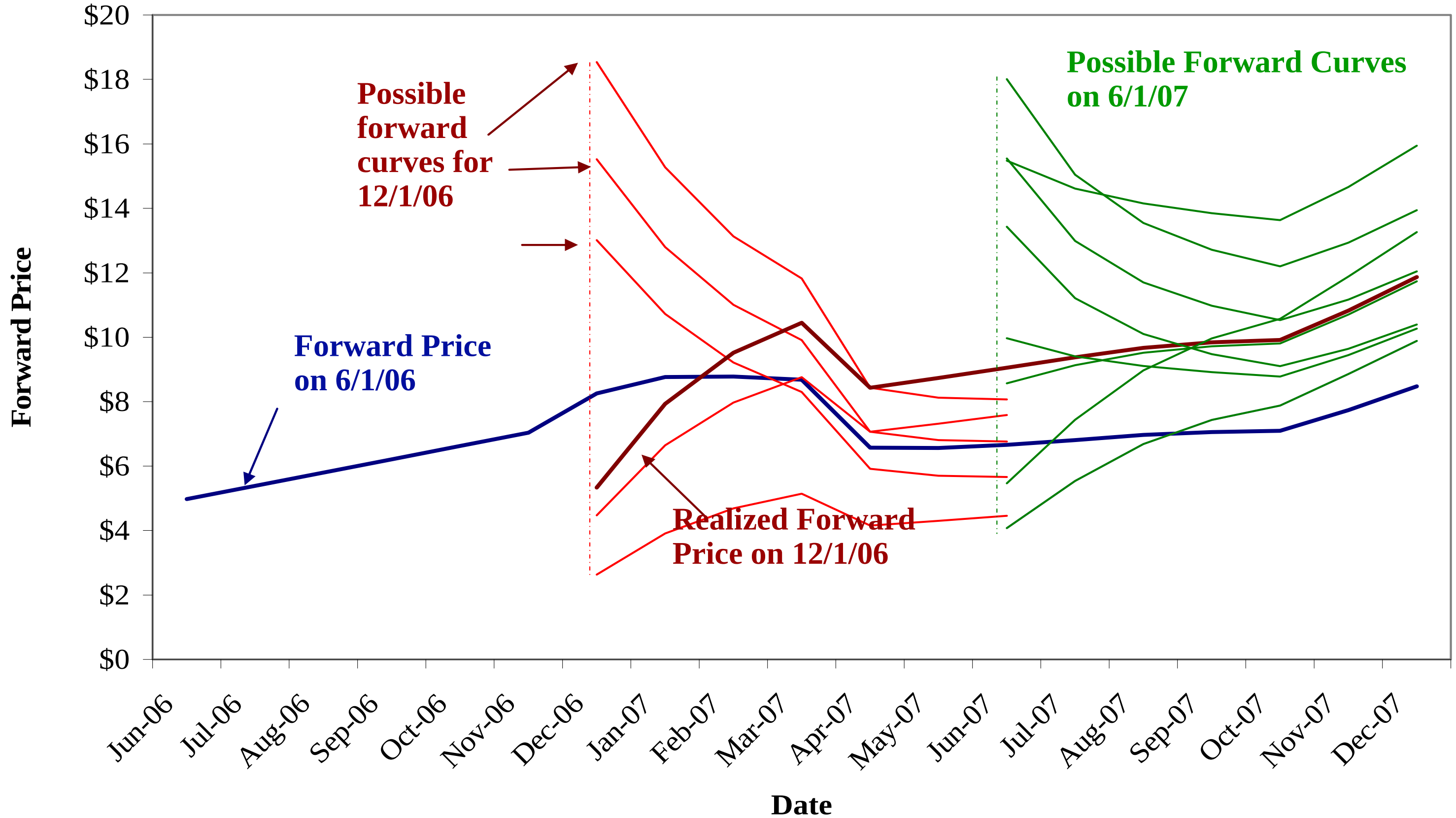
$$dw_{S,t} = -\eta w_{S,t} dt + \beta_S dz_t^S$$

$$dw_{L,t} = \beta_L dz_t^L$$

References: 1) Chapter 8 in Clewlow, L., and C. Strickland (2000): "Energy Derivatives: Pricing and Risk Management." 2) Electric Power Research Institute (EPRI) Technical Brief WO3581

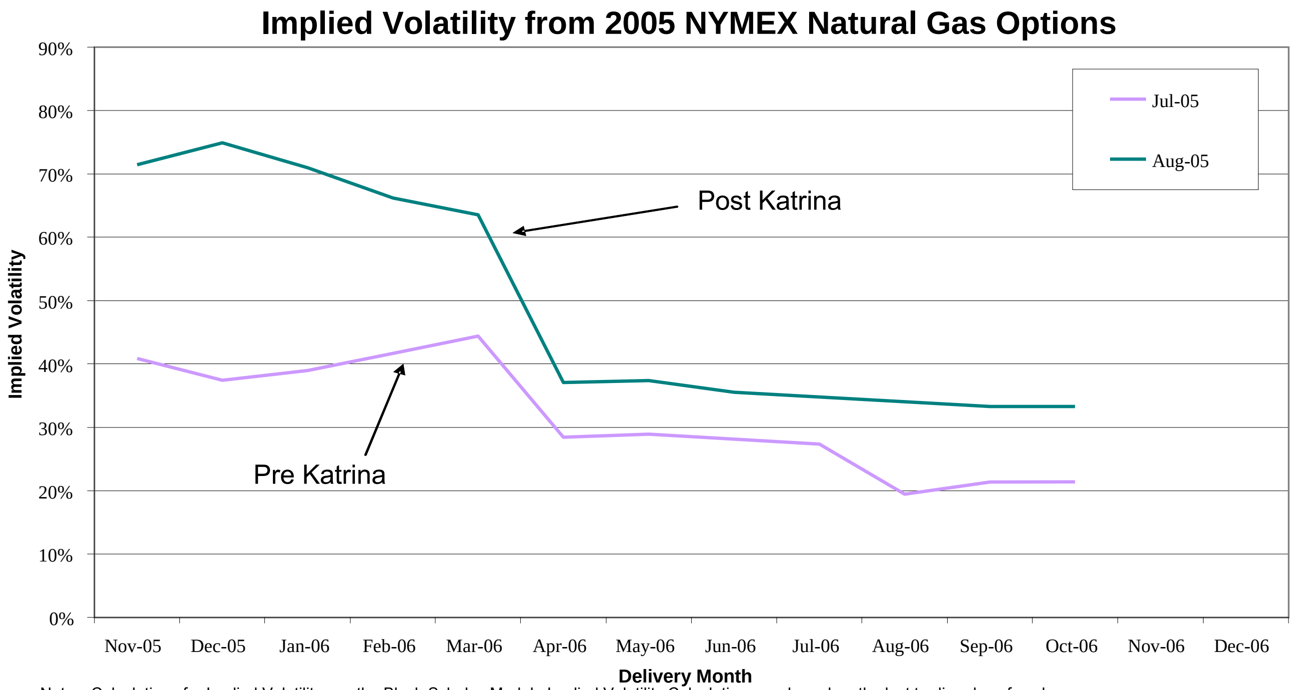
Evolution of Forward Curves

The volatility function can be used to repeatedly perturb the forward price curve in order to create simulations of what forward curves might prevail in the future.



Updating Costs and Risks Over Time

Prices and volatility both change over time. Periodically updating volatility reveals shifting market conditions that may affect desired hedging decisions.



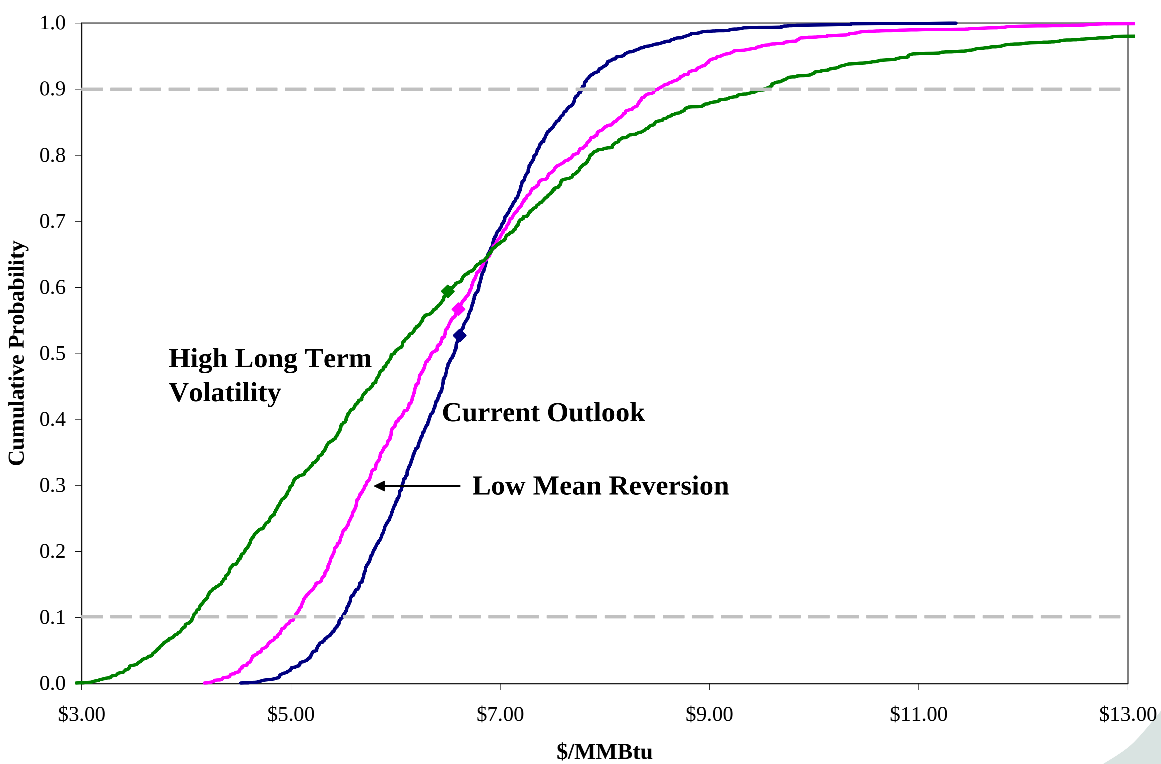
Notes: Calculations for Implied Volatility use the Black-Scholes Model. Implied Volatility Calculations are based on the last trading day of each calendar month, only including options with a positive volume.

Impact of the Two-Factor Model Parameters on Purchasing Strategy

Changes in volatilities (and forward curves) will cause initial cost distribution results to shift.

- No material impact on expected total costs from increase in volatility alone
- But procurement cost distribution becomes much wider
- And collars become more expensive (may raise expected total costs slightly)

Annual Model Results



Potential Extensions

In addition to the energy price volatility modeling described above, several other factors affecting portfolio risk and asset value are stochastic:

- Congestion
- Capacity prices
- RECs

Correlations between uncertain variables may be important to certain resources (e.g., gas-fired power plants).

And, long term volatility may change over time, e.g., when/if regulatory rules change (CO_2).

These extensions, as well as mark-to-market exposure analysis, can be accommodated with similar techniques.